CSIR/NET/JRF PHYSICAL SCIENCES

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II. **Classical Mechanics**

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III. **Electromagnetic Theory**

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IV. **Quantum Mechanics**

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V. **Thermodynamic and Statistical Physics**

Laws of thermodynamics and their consequences. Thermodynamic potentials, Maxwell relations, chemical potential, phase equilibria. Phase space, micro- and macro-states. Micro-canonical, canonical and grand-canonical ensembles and partition functions. Free energy and its connection with thermodynamic quantities. Classical and quantum statistics. Ideal Bose and Fermi gases. Principle of detailed balance. Blackbody radiation and Planck's distribution law.

VI. **Electronics and Experimental Methods**

Semiconductor devices (diodes, junctions, transistors, field effect devices, homo- and hetero-junction devices), device structure, device characteristics, frequency dependence and applications. Opto-electronic devices (solar cells, photo-detectors, LEDs). Operational amplifiers and their applications. Digital techniques and applications (registers, counters, comparators and similar circuits). A/D and D/A converters. Microprocessor and microcontroller basics. Data interpretation and analysis. Precision and accuracy. Error analysis, propagation of errors. Least squares fitting,

PART 'B' ADVANCED

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Green's function. Partial differential equations (Laplace, wave and heat equations in two and three dimensions). Elements of computational techniques: root of functions, interpolation, extrapolation, integration by trapezoid and Simpson's rule, Solution of first order differential equation using Runge Kutta method. Finite difference methods. Tensors. Introductory group theory: SU(2), O(3).

II. **Classical Mechanics**

Dynamical systems, Phase space dynamics, stability analysis. Poisson brackets and canonical transformations. Symmetry, invariance and Noether's theorem. Hamilton-Jacobi theory.

III. **Electromagnetic Theory**

Dispersion relations in plasma. Lorentz invariance of Maxwell's equation. Transmission lines and wave guides. Radiation-from moving charges and dipoles and retarded potentials.

IV. **Quantum Mechanics**

Spin-orbit coupling, fine structure. WKB approximation. Elementary theory of scattering: phase shifts, partial waves, Born approximation. Relativistic quantum mechanics: Klein-Gordon and Dirac equations. Semi-classical theory of radiation.

V. **Thermodynamic and Statistical Physics**

First- and second-order phase transitions. Diamagnetism, paramagnetism, and ferromagnetism. Ising model. Bose-Einstein condensation. Diffusion equation. Random walk and Brownian motion. Introduction to nonequilibrium processes.

VI. **Electronics and Experimental Methods**

Linear and nonlinear curve fitting, chi-square test. Transducers (temperature, pressure/vacuum, magnetic fields, vibration, optical, and particle detectors). Measurement and control. Signal conditioning and recovery. Impedance matching, amplification (Op-amp based, instrumentation amp, feedback), filtering and noise reduction, shielding and grounding. Fourier transforms, lock-in detector, box-car integrator, modulation techniques. High frequency devices (including generators and detectors).

VII. **Atomic & Molecular Physics**

Quantum states of an electron in an atom. Electron spin. Spectrum of helium and alkali atom. Relativistic corrections for energy levels of hydrogen atom, hyperfine structure and isotopic shift, width of spectrum lines, LS & JJ couplings. Zeeman, Paschen-Bach & Stark effects. Electron spin resonance. Nuclear magnetic resonance, chemical shift. Frank-Condon principle. Born-Oppenheimer approximation. Electronic, rotational, vibrational and Raman spectra of diatomic molecules, selection rules. Lasers: spontaneous and stimulated emission, Einstein A & B coefficients. Optical pumping, population inversion, rate equation. Modes of resonators and coherence length.

VIII. **Condensed Matter Physics**

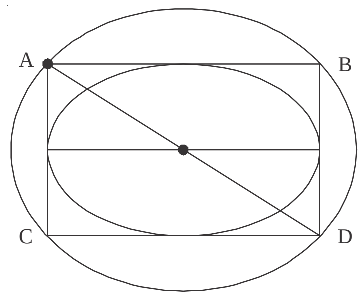
Bravais lattices. Reciprocal lattice. Diffraction and the structure factor. Bonding of solids. Elastic properties, phonons, lattice specific heat. Free electron theory and electronic specific heat. Response and relaxation phenomena. Drude model of electrical and thermal conductivity. Hall effect and thermoelectric power. Electron motion in a periodic potential, band theory of solids: metals, insulators and semiconductors. Superconductivity: type-I and type-II superconductors. Josephson junctions. Superfluidity. Defects and dislocations. Ordered phases of matter: translational and orientational order, kinds of liquid crystalline order. Quasi crystals.

IX. **Nuclear and Particle Physics**

Basic nuclear properties: size, shape and charge distribution, spin and parity. Binding energy, semiempirical mass formula, liquid drop model. Nature of the nuclear force, form of nucleon-nucleon potential, charge-independence and charge-symmetry of nuclear forces. Deuteron problem. Evidence of shell structure, single-particle shell model, its validity and limitations. Rotational spectra. Elementary ideas of alpha, beta and gamma decays and their selection rules. Fission and fusion. Nuclear reactions, reaction mechanism, compound nuclei and direct reactions. Classification of fundamental forces. Elementary particles and their quantum numbers (charge, spin, parity, isospin, strangeness, etc.). Gellmann-Nishijima formula. Quark model, baryons and mesons. C, P, and T invariance. Application of symmetry arguments to particle reactions. Parity non-conservation in weak interaction. Relativistic kinematics.

Ans-a

The diagonal of the largest square fitting in the crcle is equal to the diagonal of the circle as shown in the fig.



Let 's' is the side of the suqare. The diagonal of the suqare is given by,

$$AD = AC^2 + CD^2 \Rightarrow 2r = \sqrt{2}s \Rightarrow s = \sqrt{2}r \qquad \text{(SJ17.01)}$$

The radius of the smaller circle is

$$EF = 2r' = s = \sqrt{2}r \Rightarrow r' = \frac{r}{\sqrt{2}} \qquad \text{(SJ17.02)}$$

13. In how many ways can you place N coins on a board with N rows and N columns such that every row and every column contains exactly one coin?

a. N **b.** $N(N-1)(N-2)...2\times1$

c. N^2 **d.** N^N

Ans-b

There are N ways to distribute in rows, $(N-1)$ ways for next $(N-1)$ rows, $(N-2)$ ways for $(N-2)$ rays and so on. Thus, the total number of ways so that there shall be exactly one coin in each cell is

$$= N(N-1)(N-2)...3.2.1$$

14. A 100 m long train crosses a bridge 200m long and 20m wide bridge in 20 seconds What is the speed of the train in km/hr?

a. 45 **b.** 36

c. 54 **d.** 57.6

Ans-c

We have

$$L_T = 100m, \qquad L_S = 200m, \qquad T = 20\sec \qquad \text{(SJ17.01)}$$

The speed of the train in km/hour is

$$v = \frac{L_T + L_S}{T} = \frac{200+100}{20} = 15\,m/s = 15\times\frac{18}{5}km/hr = 54km/hr \qquad \text{(SJ17.02)}$$

15. My brthday is in January. What would be a sufficient number of questions with 'Yes/No' answers that will enable one to find my birth date?

a. 6 **b.** 3

c. 5 **d.** 2

Ans-c

In order to determine the date of brith in January, oner must ask 5 question, Let DOB is 7 Jan 2017.

1. Is the DOB an odd number?

Ans. Yes [if NO than date is even number]

35. **If the root-mean squared momentum of a particle in the ground state of a one-dimensional simple harmonic potential is P_0 then its root-mean-squared momentum in the first excited state is**

a. $p_0\sqrt{2}$ **b.** $p_0\sqrt{3}$

c. $p_0\sqrt{2/3}$ **d.** $p_0\sqrt{3/2}$

Ans-b

The expectation value of p^2 for n[th] excited state is

$$\langle p^2\rangle = \frac{m\omega\hbar}{2}(2n+1) \quad \text{(SJ17.01)}$$

For ground state, expectation value of squared of momentum operator and average moment operator are

$$\langle p_{0g}^2\rangle = \frac{m\omega\hbar}{2}(2.0+1) = \frac{m\omega\hbar}{2} \;;\; \langle p_{0g}\rangle = 0 \quad \text{(SJ17.02)}$$

The rms momentum of the ground state is,

$$p_{0rms} = \sqrt{\langle p_{0g}^2\rangle - (\langle p_{0g}\rangle)^2} = \sqrt{\frac{m\omega\hbar}{2}} = p_0 \quad \text{(SJ17.03)}$$

Similarly for first excited state, the expectation value of squared of momentum operator is

$$\langle p_{1g}^2\rangle = \frac{m\omega\hbar}{2}(2.1+1) = \frac{3m\omega\hbar}{2} \;;\; \langle p_{1g}\rangle = 0 \quad \text{(SJ17.04)}$$

The rms momentum for first excited state is

$$p_{1rms} = \sqrt{\langle p_{1g}^2\rangle - (\langle p_{1g}\rangle)^2} = \sqrt{3\frac{m\omega\hbar}{2}} = \sqrt{3}p_0 \quad \text{(SJ17.05)}$$

36. **Consider a potential barier A of height V_0 and wdth b, and another potential barrier B of height $2V_0$ and the same width b. The ratio T_A / T_B of tunneling probabilities T_A & T_B through barriers A and B respectively, for a particle of energy $V_0 / 100$, is best approximated by**

a. $\exp\left[\left(\sqrt{1.99}-\sqrt{0.99}\right)\sqrt{8mV_0b^2/\hbar^2}\right]$ **b.** $\exp\left[\left(\sqrt{1.98}-\sqrt{0.98}\right)\sqrt{8mV_0b^2/\hbar^2}\right]$

c. $\exp\left[\left(\sqrt{2.99}-\sqrt{0.99}\right)\sqrt{8mV_0b^2/\hbar^2}\right]$ **d.** $\exp\left[\left(\sqrt{2.98}-\sqrt{0.98}\right)\sqrt{8mV_0b^2/\hbar^2}\right]$

Ans-a

The transmission probability through rectangular barrier of length b is given by

$$T = Ae^{-2\beta b} \quad \text{(SJ17.01)}$$

where A is constant, $b =$ width and β is defined as

$$\beta = \sqrt{\frac{2m}{\hbar^2}(V-E)} \quad \text{(SJ17.02)}$$

we have

$$V_a = V_0, V_b = 2V_0, E = \frac{V_0}{100} \quad \text{(SJ17.03)}$$

The ratio of transmission probability is

$$\frac{T_A}{T_B} = \frac{Ae^{-2\beta_a b}}{Ae^{-2\beta_b b}} = e^{-2(\beta_b-\beta_a)b} = Exp\left(\left(\sqrt{1.99}-\sqrt{0.99}\right)\sqrt{8mV_0b^2/\hbar^2}\right) \quad \text{(SJ17.04)}$$

c. $\left\{\begin{pmatrix}1 & 0 & a\\0 & 1 & b\\0 & 0 & 1\end{pmatrix} : a,b \in R\right\}$

d. $\left\{\begin{pmatrix}1 & a & 0\\b & 1 & 0\\0 & 0 & 1\end{pmatrix} : a,b \in R\right\}$

Ans-c

Group of matrix satisfies four property

(a) The product of two matrix $AB \in G.$

(b) Associativity $(AB)C = A(BC)$

(c) Identity matrix

(d) The matrix must be invertible i.e. A exist.

The upper triangle matrix in general form the group of matrix

Let us define the matrices A and B as,

$$A = \begin{bmatrix}1 & 0 & a\\0 & 1 & b\\0 & 0 & 1\end{bmatrix}, \quad B = \begin{bmatrix}1 & 0 & r\\0 & 1 & s\\0 & 0 & 1\end{bmatrix} \qquad \text{(SJ17.01)}$$

The product of the two matrices is given by,

$$AB = \begin{bmatrix}1 & 0 & a\\0 & 1 & b\\0 & 0 & 1\end{bmatrix}\begin{bmatrix}1 & 0 & r\\0 & 1 & s\\0 & 0 & 1\end{bmatrix} = \begin{bmatrix}1 & 0 & a+r\\0 & 1 & b+s\\0 & 0 & 1\end{bmatrix} \qquad \text{(SJ17.02)}$$

The product also $AB \in G.$ Similarly the two matrices are defined as,

$$A = \begin{bmatrix}1 & a & 0\\0 & 1 & b\\0 & 0 & 1\end{bmatrix}, \quad B = \begin{bmatrix}1 & r & 0\\0 & 1 & s\\0 & 0 & 1\end{bmatrix} \qquad \text{(SJ17.03)}$$

The product of the two matrices is given by,

$$AB = \begin{bmatrix}1 & a & 0\\0 & 1 & b\\0 & 0 & 1\end{bmatrix}, B = \begin{bmatrix}1 & r & 0\\0 & 1 & s\\0 & 0 & 1\end{bmatrix} = \begin{bmatrix}1 & a+r & s\\0 & 1 & b+s\\0 & 0 & 1\end{bmatrix} \qquad \text{(SJ17.04)}$$

The product AB does not belong to the group. Rest of the properties are satisfied by both matrices. Hence the correct option is c.

55. The Lagrangian of a free relativistic particle(in one dimension) of mass is given by $L = -m\sqrt{1 - x^2}$ where $\dot{x} = dx/dt$. If such a particle is acted upon by a constant force in the direction of the motion, the phase space trajectories obtained from the corresponding Hamiltonian are

a. ellipses

b. cycloids

c. hyperbolas

d. parabolas

Ans-c

We have

$$L = -m\sqrt{1 - \dot{x}^2} - V(x) \qquad \text{(SJ17.01)}$$

The Hamiltonian for the system is defined as

$$H = \dot{x}p - L \qquad \text{(SJ17.02)}$$

The conjugate momenta is given by,

$$p = \frac{\partial L}{\partial \dot{x}} = \frac{m\dot{x}}{\sqrt{1 - x^2}} \Rightarrow \dot{x} = \frac{p}{\sqrt{m^2 + p^2}} \qquad \text{(SJ17.03)}$$

Ans-c

We have

$$B_{21} = 2.1\times10^{19}\, m^2\omega^{-1}s^{-1}, \lambda = 3000\text{Å} \quad \text{(SJ17.01)}$$

The ratio of Einstein coefficient is

$$\frac{A_{21}}{B_{21}} = \frac{8\pi h\nu^3}{c^3} \Rightarrow A_{21} = \frac{8\pi h\nu^3}{c^3} B_{21} \quad \text{(SJ17.02)}$$

Inserting eq. (SJ17.01) in eq. (SJ17.02), we obtain

$$A_{21} = \frac{8\times3.14\times6.6\times10^{-34}\times2.1\times10^{19}}{\left(3000\times10^{-10}\right)^3} = \frac{8\times3.14\times6.6\times2.1}{27}\times10^{6} \quad \text{(SJ17.03)}$$

The life time of the excited state is approximately

$$\tau = \frac{1}{A_{21}} = \frac{27}{8\times3.14\times6.6\times2.1}\times10^{-6} = 80\,ns \quad \text{(SJ17.04)}$$

75. If the binding energies of the electron in the K and L shells of silver atom are 25.4 keV and 3.34 keV, respectively, then the kinetic energy of the Auger electron will be approximately

a. 22 keV **b.** 9.3keV

c. 10.5 keV **d.** 18.7 keV

Ans-d

The kinetic energy of an ejected Augar electron is equal to the energy $h\nu$ of the charecterstic X-ray minus the binding energy of the ejected electron in the respective shell

$$K.E_{auger} = h\nu - BE_e \quad \text{(SJ17.01)}$$

Silver atom is bombarded with k_{ga} radiation from Tungsten (energy = 59.1 KeV) (z = 44) with

$$E_k = 254keV, \qquad E_L = 3.34keV \quad \text{(SJ17.02)}$$

$$E_{k\beta}(Ag) = 24.9keV, \qquad E_{k\alpha}(Ag) = 22.1keV \quad \text{(SJ17.03)}$$

The kinetic energy of the Augar electron ejected from the L shell by k_β x-rays is

$$k.E = 24.9 - 3.34 = 21.56keV \quad \text{(SJ17.04)}$$

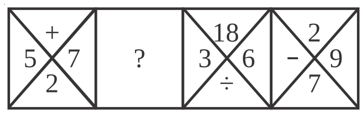
by k_α x-rays is

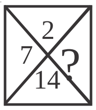
$$k.E = 22.1 - 3.34 = 18.76keV \quad \text{(SJ17.05)}$$

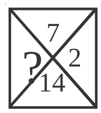
December-2016

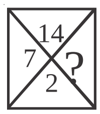
Part-A

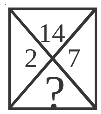
1. **Find out the missing pattern.**



a. 

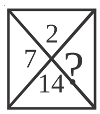
b. 

c. 

d. 

Ans-b

One can see that the missing bracket must contain sign of multiplication and the product of the two numbers are multiplied to give the third term in lower triangle of the suqare i.e.,



2. **Seeds when soaked in water gain about 20% by weight and 10% by volume. By what factor does the density increase?**

a. 1.20 **b.**1.10

c. 1.11 **d.**1.09

Ans-d

We have,

$$M' = m + 20\% of\ m = 1.2m;\ V' = v + 10\%\ of\ v = 1.1v \quad \text{(SD16.01)}$$

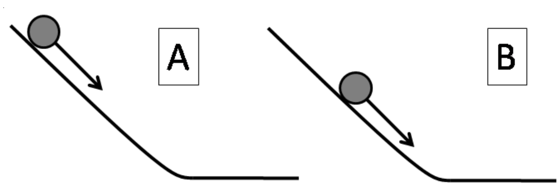
Let 'm' and 'v' are the initial mass and volume, respectively. The initial density is defined as,

$$\rho = M / V \quad \text{(SD16.02)}$$

The new density of the seeds is

$$\rho' = \frac{M'}{V'} = \frac{1.2m}{1.1v} = 1.09\frac{m}{v} = 1.09\rho \quad \text{(SD16.03)}$$

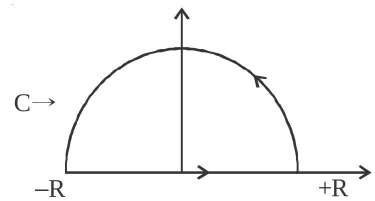
3. **Retarding frictional force *f*, on a moving ball, is proportional to its velocity V. Two identical balls roll down identical slopes (A & B) from different heights. Compare the retarding forces and the velocities of the balls at the bases of the slopes.**



a. $f_A > f_B\ ; V_A > V_B$ **b.** $f_A > f_B\ ; V_B > V_A$

c. $f_B > f_A\ ; V_B > V_A$ **d.** $f_B > f_A\ ; V_A > V_B$

Ans-a



Residue of the function f(z) at the pole $z = +i\sqrt{2}$ is,

$$\operatorname{Re} sidue = \lim_{z \to i\sqrt{2}} \left(z - i\sqrt{2}\right) f\left(z\right) = \frac{e^{ik\left(i\sqrt{2}\right)}}{i2\sqrt{2}} = \frac{e^{-\sqrt{2}k}}{2\sqrt{2}} \qquad \text{(SD16.04)}$$

Applying the cauchy's integral formulae, we obtain

$$\oint \frac{e^{ikz} dz}{z^2 + 2} = 2\pi i \operatorname{Re} si\left(F\left(z\right)\right) = \frac{2\pi i e^{-\sqrt{2}k}}{i2\sqrt{2}} = \frac{\pi e^{-\sqrt{2}k}}{\sqrt{2}} \qquad \text{(SD16.05)}$$

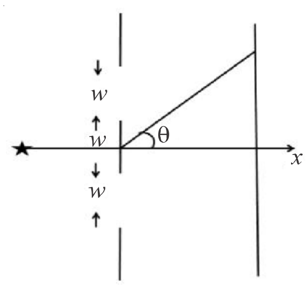
and also,

$$I = \oint \frac{e^{ikz} dz}{z^2 + 2} = \int_{-R}^{R} \frac{e^{ikx} dx}{x^2 + 2} + \oint_C \frac{e^{ikz} dz}{z^2 + 2} \Rightarrow \int_{-\infty}^{\infty} \frac{e^{ikx} dx}{x^2 + 2} = \oint \frac{e^{ikz} dz}{z^2 + 2} = \frac{\pi e^{-\sqrt{2}k}}{\sqrt{2}} \qquad \text{(SD16.06)}$$

When $R \to \infty$, we obtain required integral and the second integral along the curve vanishes, as

$$\int F\left(z\right) dz \Rightarrow \frac{1}{z^2} \to 0 \qquad \text{(SD16.07)}$$

30. A screen has two slits, each of width w with their centres at a distance $2w$ apart. It is illuminated by a monochromatic plane wave travelling along the x-axis.



The intensity of the interference pattern, measured on a distant screen, at an angle $\theta = n\lambda / w$ to the x-axis is

a. zero for $n = 1,2,3...$

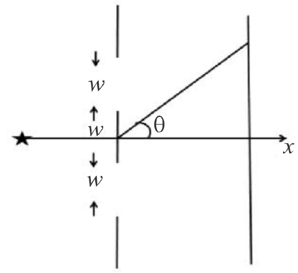
b. maximum for $n = 1,2,3...$

c. maximum for $n = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}...$

d. zero for only $n = 0$ only

Ans-a

A screen has two slits, each of width w with their centres at a distance $2w$ apart. It is illuminated by a monochromatic plane wave travelling along the x-axis.



Assuming that the interference takes place only between light reflected by the bottom surface of the top plate and the top surface of bottom plate, the distance *d* is closest to

a. 12 μm **b.** 24 μm

c. 60 μm **d.** 120 μm

Ans-d

The constructive interference for such a system are obtained, if it satisfies the condition,

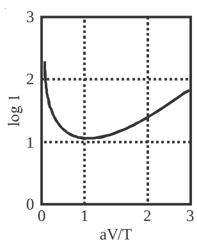
$$2d = \left(n + (1/2)\right)\lambda \quad \text{(SD16.01)}$$

where, 'n' are the integer number. The distance between two plates are

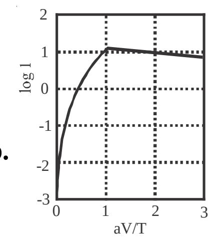
$$2d = \frac{\lambda}{2} \Rightarrow d = \frac{\lambda}{4} = \frac{495}{4}\mu m \approx 120\mu m \quad \text{(SD16.02)}$$

42. **The *I-V* characteristics of a device can be expressed as $I = I_S\left[\exp\left(\frac{aV}{T}\right) - 1\right]$, where *T* is the temperature and *a* and I_S are constants independent of *T* and *V*. Which one of the following plots is correct for a fixed applied voltage *V*?**

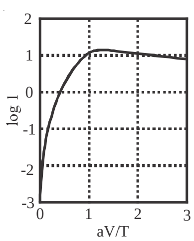
a.



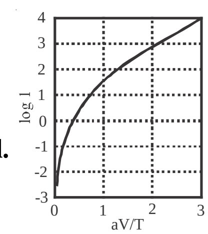
b.



c.



d.



Ans-d

The *I-V* characteristics of a device can be expressed as

$$I = I_S\left[\exp\left(\frac{aV}{T}\right) - 1\right], \quad \text{(SD16.01)}$$

where *T* is the temperature and *a* and I_S are constants independent of *T* and *V*. The eq. (SD16.01) is re-expressed as

$$I = I_s\left[\exp\left(\frac{av}{T}\right) - 1\right] \Rightarrow I = I_s\left[\exp(x) - 1\right] \Rightarrow \ln I = \ln I_s + \ln\left(\exp(x) - 1\right) \Rightarrow \ln I = \ln I_s + x \quad \text{(SD16.02)}$$

where, we have used, $x = aV/T$. The eq. (SD16.02),for a fixed applied voltage *V*, is correctly shown in the option 4.

43. **The active medium in a blue LED (light emitting diode) is a $Ga_xIn_{1-x}N$ alloy. The band gaps of GaN and InN are 3.5eV and 1.5eV respectively. If the band gap of $Ga_xIn_{1-x}N$ varies approximately linearly with *x*, the value of *x* required for the emission of blue light of wavelength 400 nm is (take $hc \approx 1200$ eV-nm)**

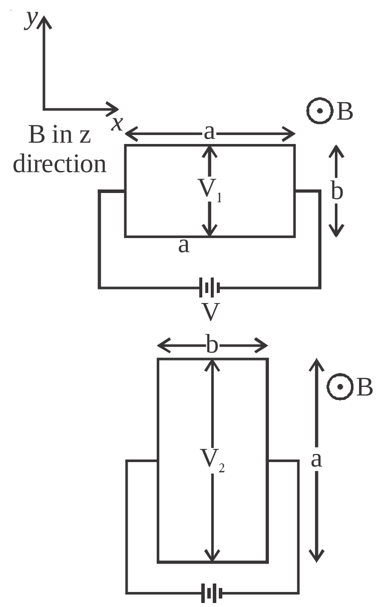
a. 0.95 **b.** 0.75

c. 0.50 **d.** 0.33

Ans-b

We have,

$$E_{GaN} = 3.5eV, E_{InN} = 1.5eV \quad \text{(SD16.01)}$$



Let $v_d, B\,\&\,d$ are the drift velocity of electron, applied magnetic field and d is the width along y direction, respectively. The drift velocity of the electron is inversely proportional to the width along x-direction $\left(v_d \propto 1/x\right)$. The hall voltage across the plates is defined as,

$$V_H = V_d Bd, \qquad \text{(SD16.01)}$$

The ratio of the hall voltage developed across the plate in two different orientation is

$$\frac{V_{H_2}}{V_{H_1}} = \frac{v_{d_2} Bd_2}{v_{d_1} Bd_2} = \frac{2}{1} \Rightarrow \frac{a^2}{b^2} \Rightarrow 2 \Rightarrow \frac{a}{b} = \frac{\sqrt{2}}{1} \Rightarrow a:b = \sqrt{2}:1 \qquad \text{(SD16.02)}$$

Where, we have used,

$$v_{d_1} = \frac{k}{a}, d_1 = b \; ; \; v_{d_2} = \frac{k}{b}; \; d_2 = a \qquad \text{(SD16.03)}$$

June-2016
Part-A

1. **"My friend Raju has more than 1000 books", said Ram. "Oh no, he has less than 1000 books", said Shyam. "Well, Raju certainly has at least one book", said Geeta. If only one of these statements is true, how many books does Raju have?**

a. 1 **b.** 1000

c. 999 **d.** 1001

Ans-b

Let Ram has books $x > 1000$, shyam has books, $y < 1000$, and Raju has books $z > 1$. The possible distribution of books are shown below.

1000	1001	999
F	T	F
F	F	T
T	T	T

Since, only one statement is true, hence raju has 1000 books.

2. **Of the following, which is the odd one out?**

a. Cone **b.** Torus

c. Sphere **d.** Ellipsoid

Ans- a or b

Cone in the odd one, because it one side in plane.

3. **An infinite number of identical circular discs each of radius 1/2are tightly packed such that the centres of the discs are at integer values of coordinates x and y. The ratio of the area of the uncovered patches to the total area is**

a. $1-\pi/4$ **b.** $\pi/4$

c. $1-\pi$ **d.** π

Ans- a

4. **It takes 5 days for a steamboat to travel from A to B along a river. It takes 7 days to return from B to A. How many days will it take for a raft to drift from A to B (all speeds stay constant)?**

a. 13 **b.** 35

c. 6 **d.** 12

Anc-b

Let u and v are the speed of boat and speed of water current, respectively. Let D is the distance between the points between A and B. The time taken by the boat in travelling A to B in upstream is

$$T_{up} = 7days = \frac{D}{u-v} \Rightarrow u-v = \frac{D}{7} \qquad \text{(SJ16.01)}$$

Similarly, the time taken by the boat in travelling B to A in downstream is

$$T_{down} = 5days = \frac{D}{u+v} \Rightarrow u+v = \frac{D}{5} \qquad \text{(SJ16.02)}$$

Adding the eq. (SJ16.01) and eq. (SJ16.02), and solving for velocity of water current, one obtains

$\psi(x)=\sqrt{\frac{2}{L}}\left(\frac{3}{5}\sin\left(\frac{2\pi x}{L}\right)+\frac{4}{5}\sin\left(\frac{4\pi x}{L}\right)\right)$. If its energy is measured, the possible outcomes and the average value of energy are, respectively

a. $\frac{h^2}{2mL^2},\frac{2h^2}{mL^2}$ and $\frac{73}{50}\frac{h^2}{mL^2}$

b. $\frac{h^2}{8mL^2},\frac{2h^2}{mL^2}$ and $\frac{19}{40}\frac{h^2}{mL^2}$

c. $\frac{h^2}{2mL^2},\frac{2h^2}{mL^2}$ and $\frac{19}{10}\frac{h^2}{mL^2}$

d. $\frac{h^2}{8mL^2},\frac{2h^2}{mL^2}$ and $\frac{73}{200}\frac{h^2}{mL^2}$

Ans-a

The state of a particle of mass in a one-dimensional rigid box in the interval 0 to L is given by the normalised wavefunction,

$$\psi(x)=\sqrt{\frac{2}{L}}\left(\frac{3}{5}\sin\left(\frac{2\pi x}{L}\right)+\frac{4}{5}\sin\left(\frac{4\pi x}{L}\right)\right)=\frac{3}{5}|\phi_2\rangle+\frac{4}{5}|\phi_4\rangle \qquad \text{(SJ16.01)}$$

If a measurement is carried out on the system, we would obtain $\langle E_n\rangle=\frac{n^2\pi^2\hbar^2}{2ma^2}$ with a corresponding probability

$$P_n\langle E_n\rangle=|\psi_n|\phi\rangle|^2. \qquad \text{(SJ16.02)}$$

Since the initial wavefunction contain only two eigenstates of $\hat{H}$, $\phi_2(x)$ & $\phi_4(x)$, the results of the energy measurement along with the corresponding probabilities are, for the state $\phi_2(x)$,

$$E_2=\langle\phi_2|\hat{H}|\phi_2\rangle=\frac{4\pi^2\hbar^2}{2mL^2}=\frac{h^2}{2mL^2}\ ;\ P_2(E_2)=|\langle\phi_2|\psi\rangle|^2=\frac{9}{25} \qquad \text{(SJ16.03)}$$

and for state $\phi_4(x)$,

$$E_4=\langle\phi_4|\hat{H}|\phi_4\rangle=\frac{16\pi^2\hbar^2}{2ma^2}=\frac{2h^2}{mL^2};P_4(E_4)=|\langle\phi_4|\psi\rangle|^2=\frac{16}{25} \qquad \text{(SJ16.04)}$$

The average energy is

$$E=\sum_n P_nE_n=\frac{9}{25}E_2+\frac{16}{25}E_4=\frac{9h^2}{50mL^2}+\frac{64}{50mL^2}=\frac{73}{50}\frac{h^2}{2mL^2} \qquad \text{(SJ16.05)}$$

31. A magnetic field B is in the region $x>0$ and zero elsewhere. A rectangular loop, in the xy-plane, of sides l (along the x-direction) and h (along the y-direction) is inserted into the $x>0$ region from the $x<0$ region at a constant velocity $v=v\hat{x}$. Which of the following values of l and h will generate the largest EMF?

a. l = 8, h = 3

b. l = 4, h = 6

c. l = 6, h = 4

d. l = 12, h = 2

Ans-b

The induced emf associated with the motion of rectangular loop in the presence of magnetic field is given by

$$\varepsilon=-\frac{d\phi}{dt}=-\frac{d}{dt}(BA)=-B\frac{dA}{dt}=-Bh\frac{dx}{dt}=-Bhv \qquad \text{(SJ16.01)}$$

Where, B is the magnetic field, A is the area of rectangular loop, h is the width of rectangular loop and v is the velocity of loop. The induced emf will be maximum for higher value of width h. Since h is maximum in option (2), so $(\varepsilon)_{max}$ is maximum in this case.

32. The x- and z-components of a static magnetic field in a region are $B_x=B_0(x^2-y^2)$ and $B_z=0$, respectively. Which of the following solutions for its y-component is consistent with the Maxwell equations?

Ans-b

We have,

$$f(x)=\delta(x)+\sum_{n=1}^{\infty}\frac{d^n}{dx^n}\delta(x) \qquad \text{(SJ16.01)}$$

The fourier transform of

$$f(k)=\int dxe^{ikx}f(x)=\int dxe^{ikx}\delta(x)+\sum_{n=1}^{\infty}\int\ dxe^{-ikx}\frac{d^n}{dx^n}\delta(x) \qquad \text{(SJ16.02)}$$

Let us compute the value of first and second term in the eq. (SJ16.02), we get

$$\int dxe^{ikx}\delta(x)=e^{-k0}=1, \int\ dxe^{-ikx}\frac{d^n}{dx^n}\delta(x)=(-1)^n(ik)^n \qquad \text{(SJ16.03)}$$

Inserting eq. (SJ16.03) in eq. (SJ16.02), one obtains

$$F(k)=1+\sum_{n=1}^{\infty}(-1)^n(ik)^n=1-ik+(ik)^2-(ik)^3+(ik)^4+..$$

$$=1-ik\left[1-(ik)+(ik)^2..\right]=1-\frac{ik}{1+ik}=\frac{1}{1+ik}\Rightarrow f(k)=\frac{1}{1+ik} \qquad \text{(SJ16.04)}$$

49. The integral equation

$$\phi(\mathbf{x,t})=\lambda\int \mathbf{dx'dt'}\int\frac{\mathbf{d\omega dk}}{(2\pi)^2}\frac{\mathbf{e}^{-\mathbf{ik}(\mathbf{x}-\mathbf{x}')+\mathbf{i}\omega(\mathbf{t}-\mathbf{t}')}}{\omega^2-\mathbf{k}^2-\mathbf{m}^2+\mathbf{i}\hat{\mathbf{I}}}\phi^3(\mathbf{x',t'})$$

is equivalent to the differential equation

a. $\left(\frac{\partial^2}{\partial t^2}+\frac{\partial^2}{\partial x^2}-m^2+i\in\right)\phi(x,t)=-\frac{1}{6}\lambda\phi^3(x,t)$

b. $\left(\frac{\partial^2}{\partial t^2}-\frac{\partial^2}{\partial x^2}+m^2-i\in\right)\phi(x,t)=\lambda\phi^2(x,t)$

c. $\left(\frac{\partial^2}{\partial t^2}-\frac{\partial^2}{\partial x^2}+m^2-i\in\right)\phi(x,t)=-3\lambda\phi^2(x,t)$

d. $\left(\frac{\partial^2}{\partial t^2}-\frac{\partial^2}{\partial x^2}+m^2-i\in\right)\phi(x,t)=-\lambda\phi^3(x,t)$

Ans-d

50. A canonical transformation $(\mathbf{q,p})\rightarrow(\mathbf{Q,P})$ is made through the generating function $\mathbf{F(q,p)=q^2P}$ on the Hamiltonian

$$\mathbf{H(q,p)=\frac{p^2}{2\alpha q^2}+\frac{\beta}{4}q^4}$$

where α and β are constants. The equations of motion for (Q, P) are

a. $\dot{Q}=P/\alpha$ and $\dot{P}=-\beta Q$

b. $\dot{Q}=4P/\alpha$ and $\dot{P}=-\beta Q/2$

c. $\dot{Q}=P/\alpha$ and $\dot{P}=-\frac{2P^2}{Q}-\beta Q$

d. $\dot{Q}=2P/\alpha$ and $\dot{P}=-\beta Q$

Ans-b

We have,

$$H(q,p)=\frac{p^2}{2\alpha q^2}+\frac{\beta}{4}q^4 \qquad \text{(SJ16.01)}$$

The hamiltonian in the system (q,p) is transformed through canonical transformation to a new system (Q, P) by using the legendre transformation, genrating function,

$$F_2(q,P)=q^2P \qquad \text{(SJ16.02)}$$

The position and momenta co-ordinates in the new system are obtained from Hamiltonian equation of motion i.e.,

$$p=\frac{\partial F_2}{\partial q}=2qP;\ Q=\frac{\partial F_2}{\partial P}=q^2 \qquad \text{(SJ16.03)}$$

74. Let E_S denote the contribution of the surface energy per nucleon in the liquid drop model. The ratio $E_S\left({}^{27}_{13}AI\right) : E_S\left({}^{64}_{30}Zn\right)$ is

a. 2:3 **b.** 4:3

c. 5:3 **d.** 3:2

Ans-b

Let E_S denote the contribution of the surface energy per nucleon in the liquid drop model and is equal to

$$\frac{E_S}{A} \propto 4\pi R^2 \Rightarrow \frac{E_S}{A} = kA^{2/3} \qquad \text{(SJ16.01)}$$

where, we have used,

$$R = R_0 A^{1/3} \qquad \text{(SJ16.02)}$$

The energy of $E_S\left({}^{27}_{13}AI\right) \& E_S\left({}^{64}_{30}Zn\right)$ are

$$\left(\frac{E_S}{A}\right)\left({}^{27}A\ell\right) = k \quad 27^{2/3} \quad ; \quad \left(\frac{E_S}{A}\right)\left({}^{64}Zn\right) = k\ (64)^{2/3} \qquad \text{(SJ16.03a, 03b)}$$

Dividing eq. (SJ16.03a) by eq. (SJ6.03b), we get

$$\frac{E_S\left({}^{27}A_\ell\right)}{E_S\left({}^{64}Z_n\right)} = \frac{A(Zinc)}{A(A\ell)}\left(\frac{27}{64}\right)^{2/3} = \frac{64}{27} \times \frac{9}{16} = \frac{4}{3} \qquad \text{(SJ16.04)}$$

75. In the large hadron collider (LHC), two equal energy proton beams traverse in opposite directions along a circular path of length 27 km. If the total centre of mass energy of a proton-proton pair is 14 TeV, which of the following is the best approximation for the proper time taken by a proton to traverse the entire path?

a. 12 ns **b.** $1.2\mu s$

c. 1.2 ns **d.** $0.12\mu s$

Ans-a

We have,

$$S = 27\,km, E = 27TeV, c = 3\times10^8\,m/s \qquad \text{(SJ16.01)}$$

The time taken to complete one round trip is,

$$T = \frac{S}{c} = \frac{27\times10^3\,m}{3\times10^8\,s} = 90\mu s \qquad \text{(SJ16.02)}$$

The value of relative exponent is given by,

$$p = \gamma m_p c \Rightarrow \gamma = \frac{p}{m_p c} = \frac{cp}{m_p c^2} = \frac{E_p}{m_p c^2} = \frac{7\times10^{12}\,eV}{938MeV} = 7.48\times10^3 \qquad \text{(SJ16.03)}$$

The proper time taken by a proton to traverse the entire path

$$T = \gamma T_0 \Rightarrow T_0 = \frac{T}{\gamma} = \frac{90\,ns}{7.48} = 12ns \qquad \text{(SJ16.04)}$$

December-2015

Part-A

1. **A shopkeeper purchases a product for Rs. 100 and sells it making a profit of 10%. The coustomer resells it to the same shopkeeper incurring a loss of 10%. In these dealings the shopkeeper makes**

a. no profit, no loss **b.** Rs. 11

c. Re. 1 **d.** Rs. 20

Ans-b

The cost price of the object for the shopkeeper is Rs 100, the shopkeeper sells the object at profit of 10%, so the selling price of the object is

$$SP = CP + 10\%\ of\ CP = 100 + \frac{10}{100}\times 100 = Rs110 \qquad \text{(SD15.01)}$$

The customer resale that object at loss of 10%, so the selling price for the customer is

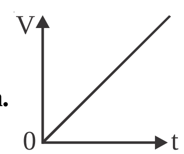
$$SP = CP - 10\%\ of\ CP = 110 - \frac{110}{100}\times 100 = Rs\,99 \qquad \text{(SD15.02)}$$

The profit of the shopkeeper is

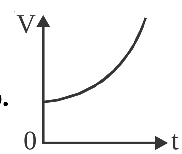
$$profit = 10(selling) + 1(reselling) = Rs11 \qquad \text{(SD15.03)}$$

2. **A vessel is partially filled with water. More water is added to it at a rate directly proportional to time $\left[\text{i.e., } \frac{dv}{dt} \propto t\right]$. Which of the following graphs depicts correctly the variation of total volume V of water with time t?**

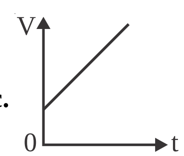
a.



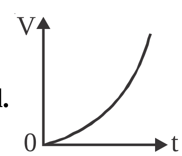
b.



c.



d.



Ans-b

We have,

$$\frac{dv}{dt} \propto t \Rightarrow v = a\frac{t^2}{2} + c \qquad \text{(SD15.01)}$$

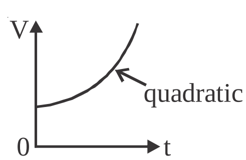
Applying the boundary condition $v(t=0) = v_0$, on the eq. (SD15.01), we get

$$v(t=0) = v_0 = 0 + c \Rightarrow c = v_0 \qquad \text{(SD15.02)}$$

Inserting eq. (SD15.02) in eq. (SD15.01), we obtain

$$v = v_0 + a_0 t^2\ ; a_0 = a/2 \qquad \text{(SD15.03)}$$

The eq. (SD15.03) is correctly represented by the curve shown in the figure given below.



b. $\ell n\sqrt{\frac{y-1}{y+1}}$

d. $\ell n\sqrt{\frac{y+1}{y-1}}$

Ans-d

We have,

$$y=\frac{1}{\tanh(x)}=\frac{Cosh(x)}{Sinh(x)}=\frac{e^{x}+e^{-x}}{e^{x}-e^{-x}}=\frac{e^{2x}+1}{e^{2x}-1} \qquad \text{(SD15.01)}$$

Simplifying the eq. (SD15.01), and solving for x, one obtains

$$e^{2x}\cdot y-y=e^{2x}+1\Rightarrow e^{2x}=\frac{1+y}{y-1}\Rightarrow x=\frac{1}{2}\ell n\left(\frac{1+y}{y-1}\right)\Rightarrow x=\ell n\sqrt{\left(\frac{1+y}{y-1}\right)} \qquad \text{(SD15.02)}$$

30. A Hermitian operator $\hat{O}$ has two normalised eigenvalues $|1\rangle$ and $|2\rangle$ with eigenvalues 1 and 2, respectively. The two states $|u\rangle = \cos\theta|1\rangle+\sin\theta|2\rangle$ and $|v\rangle = \cos\phi|1\rangle+\sin\phi|2\rangle$ are such that $\langle v|\hat{O}|v\rangle = 7/4$ and $\langle u|v\rangle = 0$. Which of the following are possible values of θ and ϕ?

a. $\theta=-\frac{\pi}{6}$ and $\phi=\frac{\pi}{3}$

b. $\theta=\frac{\pi}{6}$ and $\phi=\frac{\pi}{3}$

c. $\theta=-\frac{\pi}{4}$ and $\theta=\frac{\pi}{4}$

d. $\phi=\frac{\pi}{3}$ and $\theta=-\frac{\pi}{6}$

Ans-a

We have,

$$|u\rangle=\cos\theta|1\rangle+\sin\theta|2\rangle\ ;\ |v\rangle=\cos\phi|1\rangle+\sin\phi|2\rangle \qquad \text{(SD15.01)}$$

Now applying the orthonormality relation,we get

$$\langle u|v\rangle=\left(\cos\theta\langle 1|+\sin\theta\langle 2|\right)\left(\cos\phi|1\rangle+\sin\phi|2\rangle\right)=\cos\theta\cos\phi\langle 1|1\rangle+\sin\theta\sin\phi\langle 2|2\rangle=0$$

$$=\cos\theta\cos\phi+\sin\theta\sin\phi=0\Rightarrow\cos(\theta-\phi)=0\Rightarrow\cos(\phi-\theta)=0\Rightarrow\phi-\theta=\pi/2\ , \qquad \text{(SD15.02)}$$

This condition is satisfied by option (1) and (4) :

$$option(1):\phi-\theta=\frac{\pi}{3}+\frac{\pi}{6}=\frac{\pi}{2};\ option(2):\phi-\theta=\frac{\pi}{4}+\frac{\pi}{4}=\frac{\pi}{2} \qquad \text{(SD15.03)}$$

Applying the operator $\hat{O}$ on the wavefunction $|v\rangle$, we get

$$\hat{O}\left|\left(\cos\phi|1\rangle+\sin\phi|2\rangle\right)\right.=\cos\phi|1\rangle+2\sin\phi|2\rangle \qquad \text{(SD15.04)}$$

The expectation value of $\hat{O}$ is

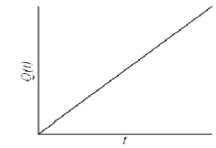
$$\langle\hat{O}\rangle=\langle v|\hat{O}|v\rangle=\left(\cos\phi\langle 1|+\sin\phi\langle 2|\right)\left(\cos\phi|1\rangle+2\sin\phi|2\rangle\right)=\cos^2\phi+2\sin^2\phi=\frac{7}{4} \qquad \text{(SD15.05)}$$

Inserting the value of $\phi=\pi/3$ in eq. (SD15.06), we obtain,

$$\cos^2\frac{\pi}{3}+2\sin^2\frac{\pi}{3}=1+\sin^2\frac{\pi}{3}=1+\left(\frac{\sqrt{3}}{2}\right)^2=1+\frac{3}{4}=\frac{7}{4} \qquad \text{(SD15.06)}$$

31. A beam of unpolarized light in a medium with dielectirc constant $\in_1$ is reflected from a plane interface formed with another medium of dielectric constant $\in_2 = 3\in_1$. The two media have identical magnetic permeability. If the angle of incidence is 60°, then the reflected light

a. is plane polarized perpendicular to the plane of incidence



Straight line
with shapeω

51. The Hermite polynomial $H_n(x)$ satisfies the differential equation $\frac{d^2H_n}{dx^2} - 2x\frac{dH_n}{dx} + 2nH_n(x) = 0$. The corresponding generating function $G(t, x) = \sum_{n=0}^{\infty} \frac{1}{n!} H_n(x) t^n$ satisfies the equation

a. $\frac{\partial^2 G}{\partial x^2} - 2x\frac{\partial G}{\partial x} + 2t\frac{\partial G}{\partial t} = 0$

b. $\frac{\partial^2 G}{\partial x^2} - 2x\frac{\partial G}{\partial x} - 2t^2\frac{\partial G}{\partial t} = 0$

c. $\frac{\partial^2 G}{\partial x^2} - 2x\frac{\partial G}{\partial x} + 2\frac{\partial G}{\partial t} = 0$

d. $\frac{\partial^2 G}{\partial x^2} - 2x\frac{\partial G}{\partial x} + 2t\frac{\partial^2 G}{\partial x \partial t} = 0$

Ans-a

The Hermite polynomial $H_n(x)$ satisfies the differential equation

$$\frac{d^2H_n}{dx^2} - 2x\frac{dH_n}{dx} + 2nH_n(x) = 0. \qquad \text{(SD15.01)}$$

The generating function

$$G(t, x) = \sum_{n=0}^{\infty} \frac{1}{n!} H_n(x) t^n \qquad \text{(SD15.02)}$$

satisfies the following differential equation i.e.,

$$\frac{\partial^2 G}{\partial x^2} - 2x\frac{\partial G}{\partial x} + 2t\frac{\partial G}{\partial t} = 0 \Rightarrow \sum_n \frac{t^n}{n!}\left(\frac{d^2H_n}{dx^2} - 2x\frac{dH_n}{dx} + 2nH_n(x)\right) = 0 \qquad \text{(SD15.03)}$$

yielding,

$$\frac{d^2H_n}{dx^2} - 2x\frac{dH_n}{dx} + 2nH_n(x) = 0. \qquad \text{(SD15.04)}$$

where, we have used,

$$\frac{\partial^2 G}{\partial x^2} = \sum \frac{1}{n!} t^n \frac{\partial^2 H_n(x)}{\partial x^2}; \quad \frac{\partial G}{\partial x} = \sum \frac{1}{n!} t^n \frac{\partial H_n(x)}{\partial x}; \; 2t\frac{\partial G}{\partial t} = \sum \frac{2n}{n!} t^n H_n(x) \qquad \text{(SD15.05)}$$

52. A dipole of moment $\vec{p}$, oscillating at frequency ω, radiates spherical waves. The vector potential at large distance is $\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} i\omega \frac{e^{ikr}}{r} \vec{p}$. To order (1/r) the magnetic field $\vec{B}$ at a point $\vec{r} = r\hat{n}$ is

a. $-\frac{\mu_0}{4\pi}\frac{\omega^2}{c}(\hat{n}\cdot\vec{p})\hat{n}\frac{e^{ikr}}{r}$

b. $-\frac{\mu_0}{4\pi}\frac{\omega^2}{c}(\hat{n}\times\vec{p})n\frac{e^{ikr}}{r}$

c. $-\frac{\mu_0}{4\pi}\omega^2 k(\hat{n}\cdot\vec{p})\,\hat{p}\frac{e^{ikr}}{r}$

d. $-\frac{\mu_0}{4\pi}\frac{\omega^2}{c}\hat{p}\frac{e^{ikr}}{r}$

Ans-b

The vector potential associated with a dipole of moment $\vec{p}$, oscillating with frequency ω, at large distance is

J-38

Which of the following statements is true?

a. Process (i) obeys all conversation laws Process

b. Process (ii) conserveses baryon number, but violates energy-momentum conservation

c. Process (iii) is not allowed by strong interactions, but is allowed by weak interactions

d. Process (iv) conserves baryon number, but violates lepton number conservation

Ans-b

In the process given below:

$$\overline{P}+n \rightarrow \pi^{-} \Rightarrow Baryon\ no: -1+1 \rightarrow 0 \qquad \text{(SD15.01)}$$

The Baryon Number is conserved. In the centre of mass frame of reference, the total momentum of $\overline{P}+n$ is zero, but then π^{-} has to be created at rest to conserve momentum which is not possible. Thus, the reaction violates the conservation of momentum.

<u>Note</u> : The combined rest masses of $\overline{P}+n$ is about $940\times 2 = 1880\,MeV$ and that of π^{-} is about 140 MeV, so lot of rest mass $(1880-140)\,MeV = 1740\,MeV$ is available to create large number of pions. But creating of only pion is not possible.

June-2015
Part-A

1. **Each of the following pairs of words hides a number, based on which you can arrange them in ascending order. Pick the correct answer:**
I. Cloth reel
J Silent wonder
K Good tone
L. Bronz rod

a. L, K, J, I **b.** I, J, K, L

c. K, L, J, I **d.** K, J, I, L

Ans- a
The hidden word in these word are:
I. Cloth reel $\rightarrow$ three $\rightarrow 3$
J. Silent Wonder $\rightarrow$ two $\rightarrow 2$
K. Good tone $\rightarrow$ one $\rightarrow 1$
L. Bronze rod $\rightarrow$ zero $\rightarrow 0$
These numbers are arranged in the following manner:

$$0 < 1 < 2 < 3 \Rightarrow L, K, J, I$$ (SJ15.01)

2. **Which of the following values is same as $2^{2^{2^2}}$?**
a. 2^6 **b.** 2^8

c. 2^{16} **d.** 2^{222}

Ans-c

The number $2^{2^{2^2}}$ is simplified as:

$$2^{2^{2^2}} = 2^{2^4} = 2^{16}$$ (SJ15.01)

3. **A 12m $\times$ 4m rectangular roof is resting on four 4 m tall thin poles. Sunlight falls on the roof at an angle of 45° from the east, creating a shadow on the ground. What will be the area of the shadow?**
a. 24 m^2 **b.** 36 m^2

c. 48 m^2 **d.** 60 m^2

Ans- c

Sunlight travels in a straight line, so the shadow will be of rectangular shape with an area $12m \times 4m = 48m^2$. The dimension 4m is due to 45° inclination.

4. **If**

$$\begin{array}{r} 2a \\ \times\ b2 \\ \hline c6 \\ 84 \\ \hline 8d6 \\ \hline \end{array}$$

The action of an operator $\hat{T}$ on $\psi(x)$ is given by

$$\hat{T}\psi(x) = \psi(x+a), \qquad \text{(SJ15.02)}$$

where a is a constant. Applying the operator T on eq. (SJ15.01), is

$$\hat{T}\phi(p) = \int T\psi(x) e^{-ipx/\hbar} dx = \int \psi(x+a) e^{-ipx/\hbar} dx$$

$$= \int \psi(x) e^{-ip(x-a)/\hbar} d(x-a) = e^{ipa/\hbar} \int \psi(x) e^{-ipx/\hbar} dx = e^{ipa/\hbar}\phi(p) \qquad \text{(SJ15.03)}$$

36. If L_i are the components of the angular momentum operator $\vec{L}$, then the operator $\sum_{i=1,2,3}\left[\left[\vec{L}, L_i\right], L_i\right]$ equals

a. $\vec{L}$ **b.** $2\vec{L}$

c. $3\vec{L}$ **d.** $-\vec{L}$

Ans-b

Let us compute the value of commutator in the following manner:

$$\sum_{i=1}^{3}\left[\left[\vec{L}, L_i\right], L_i\right] = \left[\left[\vec{L}, L_1\right], L_1\right] + \left[\left[\vec{L}, L_2\right], L_2\right] + \left[\left[\vec{L}, L_3\right], L_3\right] \qquad \text{(SJ15.01)}$$

where, the angular momentum is defined as

$$\vec{L} = \hat{i}L_1 + \hat{j}L_2 + \hat{k}L_3 \qquad \text{(SJ15.02)}$$

Let us evaluate the value of first commutator, we get

$$\left[\left[\vec{L}, L_1\right], L_1\right] = \left[-\hat{j}L_3 + \hat{k}L_2, L_1\right] = -\hat{j}\left[L_3, L_1\right] + \hat{k}\left[L_2, L_1\right] = -\hat{j}L_2 - \hat{k}L_3 \qquad \text{(SJ15.03)}$$

Where, we have used,

$$\left[\vec{L}, L_1\right] = \left[\hat{i}L_1 + \hat{j}L_2 + \hat{k}L_3, L_i\right]$$

$$= \hat{i}\left[L_1, L_1\right] + \hat{j}\left[L_2, L_1\right] + \hat{k}\left[L_3, L_1\right] = -\hat{j}\,L_3 + \hat{k}\,L_2 \qquad \text{(SJ15.04)}$$

Similarly, the second and third commutator brakets also give

$$\left[\left[\vec{L}, L_2\right], L_2\right] = -\hat{i}L_1 - \hat{k}L_3; \left[\left[\vec{L}, L_3\right], L_3\right] = -\hat{i}L_1 - \hat{j}L_2 \qquad \text{(SJ15.05)}$$

Inserting the eq. (SJ15.03) and eq.(SJ15.05) in eq. (SJ15.01), we get

$$\sum_{i=1}^{3}\left[\left[\vec{L}, L_i\right], L_i\right] = -2\hat{i}L_1 - 2\hat{j}L_2 - 2\hat{k}L_3 = -2\vec{L} \qquad \text{(SJ15.06)}$$

37. A particle moves in one dimension in the potential $V = \frac{1}{2}k(t)x^2$, where k(t) is a time dependent parameter. Then $\frac{d}{dt}\langle v\rangle$, the rate of change of the expectation value $\langle v\rangle$ of the potential energy, is

a. $\frac{1}{2}\frac{dk}{dt}\langle x^2\rangle + \frac{k}{2m}\langle xp + px\rangle$ **b.** $\frac{1}{2}\frac{dk}{dt}\langle x^2\rangle + \frac{1}{2m}\langle p^2\rangle$

c. $\frac{k}{2m}\langle xp + px\rangle$ **d.** $\frac{1}{2}\frac{dk}{dt}\langle x^2\rangle$

Ans-a

A particle moves in one dimension in the potential

$$V = \frac{1}{2}k(t)x^2, \qquad \text{(SJ15.01)}$$

where k(t) is a time dependent parameter. The rate of change of an operator $\langle A\rangle$ is given by,

$$\frac{d}{dt}\langle A\rangle = \frac{1}{i\hbar}\langle\left[A, H\right]\rangle + \frac{\partial\langle A\rangle}{\partial t} \qquad \text{(SJ15.02)}$$

Where, we have used,

$$\left[\hat{a},\, H\right] = \hbar\omega\left[\hat{a},\, \hat{a}^{+}\hat{a} + \frac{1}{2}\right] = \hbar\omega\left[\hat{a},\, \hat{a}^{+}\hat{a}\right] = \hbar\omega\hat{a}(t) \qquad \text{(SJ15.03)}$$

59. A particle of energy E scatters off a repulsive spherical potential

$$V(r) = \begin{cases} V_0 \text{ for } r < a \\ 0 \text{ for } r \geq a \end{cases}$$

where V_0 and a are positive constants. In the low energy limit, the total scattering cross-section is

$$\sigma = 4\pi a^2\left(\frac{1}{ka}\tanh ka - 1\right)^2,$$

where $k^2 = \frac{2m}{\hbar^2}(V_0 - E) > 0$. In the limit $V_0 \to \infty$ the ratio of σ to the classical scattering cross-section off a sphere of radius a is

a. 4 **b.** 3

c. 1 **d.** 1/2

Ans-a

The total scattering cross-section of a particle of energy E scatters off a repulsive spherical potential, is

$$\sigma = 4\pi a^2\left(\frac{1}{ka}\tanh ka - 1\right)^2,\; k^2 = \frac{2m}{\hbar^2}(V_0 - E) > 0 \qquad \text{(SJ15.01)}$$

In the limit $V_0 \to \infty$, it behaves like a hard sphere. The total scattering cross-section becomes in

$$\sigma_T = 4\pi a^2 \qquad \text{(SJ15.02)}$$

The geometric cross-section area of the particle is

$$\sigma_{cs} = \pi a^2 \qquad \text{(SJ15.03)}$$

The ratio of σ to the classical scattering cross-section off a sphere of radius a is

$$\frac{\sigma_T}{\sigma_{cs}} = \frac{4\pi a^2}{\pi a^2} = 4 \qquad \text{(SJ15.04)}$$

60. Two different sets of orthogonal basis vectors $\left\{\begin{pmatrix}1\\0\end{pmatrix}, \begin{pmatrix}0\\1\end{pmatrix}\right\}$ and $\left\{\frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}, \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix}\right\}$ are given for a two-dimensional real vector space. The matrix representation of a linear operator $\hat{A}$ in these bases are related by a unitary transformation. The unitary matrix may be chosen to be

a. $\begin{pmatrix}0 & -1\\1 & 0\end{pmatrix}$ **b.** $\begin{pmatrix}0 & 1\\1 & 0\end{pmatrix}$

c. $\frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\1 & -1\end{pmatrix}$ **d.** $\frac{1}{\sqrt{2}}\begin{pmatrix}1 & 0\\1 & 1\end{pmatrix}$

Ans-c

This problem is solved through options. Check all the options, but only option (3) work as follows:

$$\frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\1 & -1\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}\;;\; \frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\1 & -1\end{pmatrix}\begin{pmatrix}0\\1\end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix} \qquad \text{(SJ15.01)}$$

61. A large number N of Brownian particles in one dimension start their diffusive motion from the origin at time t = 0. The diffusion cofficient is D. The number of particles crossing a point at a distance L from the origin, per unit time, depends on L and time t as

Magic Numbers are (k = 0, 1, 2 --- etc) : 2, 8, 20, 40, 70, 112. Thus, the atoms ${}_2^4He$ & ${}_8^{16}O$ have 2, 8 magic numbers.

74. The charm quark is assigned a charm quantum number C = 1. How should the Gellmann-Nishijima formula for electric charge be modified for four flavours of quarkS?

a. $I_3+\frac{1}{2}(B-S-C)$ **b.** $I_3+\frac{1}{2}(B-S+C)$

c. $I_3+\frac{1}{2}(B+S-C)$ **d.** $I_3+\frac{1}{2}(B+S+C)$

Ans-d

The properties of the different quarks are given below:

Quark	Charge	B	S	C	I_3
Up	+2/3	1/3	0	0	1/2
Down	– 1/3	1/3	0	0	– 1/2
Charm	+2/3	1/3	0	1	0
Strange	– 1/3	1/3	–1	0	0

The formulae, which can correctly, obtains these properties is given below.

$$Q = I_3+\frac{1}{2}(B+S+C) \quad \text{(SJ15.01)}$$

75. The reaction ${}_1^2\mathbf{D}+{}_1^2\mathbf{D}\rightarrow{}_2^4\mathbf{He}+\pi^0$ cannot proceed via strong interactions because it violates the conversation of

a. angular momentum **b.** electric charge

c. baryon number **d.** isospin

Ans-d

We have,

$$ {}_1^2D+{}_1^2D\rightarrow{}_2^4He+\pi^0 \quad \text{(SJ15.01)}$$

Applying the conservation laws, one obtains

Electric charge $1+1\rightarrow 2+0$ (SJ15.01)

(Baryon No.)B $2+2\rightarrow 4+0$ (SJ15.01)

vector additon: $Angular\ momentum:\{\bar{1}+\bar{1}=0,1,2,\}\rightarrow 0+0$ (SJ15.01)

Isospin: $0+0\rightarrow 0+1$ (SJ15.01)

Since the reaction violates the Isospin conservation, hence it cannot proceed via strong interactions.

December-2014
Part-A

1. **Lunch-dinner pattern of a person of m days is given below. He has a choice of a VEG or a NON-VEG meal for his lunch/dinner**
(a) if he takes a NON-VEG lunch he will have only VEG dinner

(b) He takes NON-VEG dinner for exactlly 9 days

(c) He takes VEG lunch for exactly 15 days

(d) He takes a total of 14 NON-VEG meals What is m?

a. 18 **b.** 24

c. 20 **d.** 38

Ans-c
Let V denotes the vegetarian meal, and N is the non vegetarian meal. The sequence of lunch and dinner for 20 days are mentioned below,

sequence for lunch : (15V)(5N) = Total 20
sequence for dinner : (9N)(11V) = Total 20

2. **Two locomotives are running towards each other with speed of 60 and 40 Km/h. An object keeps on flying to and fro from the front tip of one locomotive to the front tip of other with a speed of 70 km/h. After 30 minutes, the two locomotives collide and the object cover before being crushed?**
a. 50 km **b.** 15 km

c. 35 km **d.** 10 km

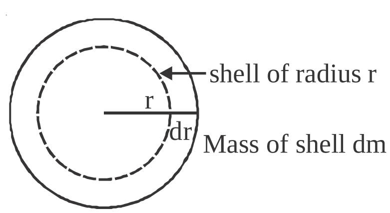
Ans-c
The distance covered by flying object in 30 min is

$$Distance = speed \times time = 70 \times \frac{1}{2} = 35\,km \qquad \text{(SD14.01)}$$

3. **A sphere is made up of very thin concentric shells of increasing radii (leaving no gaps). The mass of an arbitrarily choosen shell is**
a. Equal to the mass of the preceding shell **b.** proportional to its volume

c. proportional to its radius **d.** proportional to its surface area

Ans-d
Consider a sphere made up of large number thin spherical shells. Let us consider a thin spherical shell of radius ‘r’ and thikness ‘dr’, as shown in the figure given below.



The mass of the thin spherical shell is

$$dm = \rho dV = \rho 4\pi r^2 dr \Rightarrow dm \propto surface\,area\left(4\pi r^2\right) \qquad \text{(SD14.01)}$$

Ans-b

We have,

$$f(x)=\delta(x)+\sum_{n=1}^{\infty}\frac{d^n}{dx^n}\delta(x) \qquad \text{(SJ16.01)}$$

The fourier transform of

$$f(k)=\int dxe^{ikx}f(x)=\int dxe^{ikx}\delta(x)+\sum_{n=1}^{\infty}\int \; dxe^{-ikx}\frac{d^n}{dx^n}\delta(x) \qquad \text{(SJ16.02)}$$

Let us compute the value of first and second term in the eq. (SJ16.02), we get

$$\int dxe^{ikx}\delta(x)=e^{-k0}=1, \int \; dxe^{-ikx}\frac{d^n}{dx^n}\delta(x)=(-1)^n(ik)^n \qquad \text{(SJ16.03)}$$

Inserting eq. (SJ16.03) in eq. (SJ16.02), one obtains

$$F(k)=1+\sum_{n=1}^{\infty}(-1)^n(ik)^n=1-ik+(ik)^2-(ik)^3+(ik)^4+..$$

$$=1-ik\left[1-(ik)+(ik)^2..\right]=1-\frac{ik}{1+ik}=\frac{1}{1+ik}\Rightarrow f(k)=\frac{1}{1+ik} \qquad \text{(SJ16.04)}$$

49. The integral equation

$$\phi(\mathbf{x,t})=\lambda\int \mathbf{dx'dt'}\int\frac{\mathbf{d\omega dk}}{(2\pi)^2}\frac{\mathbf{e}^{-ik(x-x')+i\omega(t-t')}}{\omega^2-\mathbf{k}^2-\mathbf{m}^2+i\hat{I}}\phi^3(\mathbf{x',t'})$$

is equivalent to the differential equation

a. $\left(\frac{\partial^2}{\partial t^2}+\frac{\partial^2}{\partial x^2}-m^2+i\in\right)\phi(x,t)=-\frac{1}{6}\lambda\phi^3(x,t)$

b. $\left(\frac{\partial^2}{\partial t^2}-\frac{\partial^2}{\partial x^2}+m^2-i\in\right)\phi(x,t)=\lambda\phi^2(x,t)$

c. $\left(\frac{\partial^2}{\partial t^2}-\frac{\partial^2}{\partial x^2}+m^2-i\in\right)\phi(x,t)=-3\lambda\phi^2(x,t)$

d. $\left(\frac{\partial^2}{\partial t^2}-\frac{\partial^2}{\partial x^2}+m^2-i\in\right)\phi(x,t)=-\lambda\phi^3(x,t)$

Ans-d

50. A canonical transformation $(q,p)\to(Q,P)$ is made through the generating function $F(q,p)=q^2P$ on the Hamiltonian

$$\mathbf{H}(\mathbf{q,p})=\frac{\mathbf{p}^2}{2\alpha\mathbf{q}^2}+\frac{\beta}{4}\mathbf{q}^4$$

where α and β are constants. The equations of motion for (Q, P) are

a. $\dot{Q}=P/\alpha$ and $\dot{P}=-\beta Q$

b. $\dot{Q}=4P/\alpha$ and $\dot{P}=-\beta Q/2$

c. $\dot{Q}=P/\alpha$ and $\dot{P}=-\frac{2P^2}{Q}-\beta Q$

d. $\dot{Q}=2P/\alpha$ and $\dot{P}=-\beta Q$

Ans-b

We have,

$$H(q,p)=\frac{p^2}{2\alpha q^2}+\frac{\beta}{4}q^4 \qquad \text{(SJ16.01)}$$

The hamiltonian in the system (q,p) is transformed through canonical transformation to a new system (Q, P) by using the legendre transformation, genrating function,

$$F_2(q,P)=q^2P \qquad \text{(SJ16.02)}$$

The position and momenta co-ordinates in the new system are obtained from Hamiltonian equation of motion i.e.,

$$p=\frac{\partial F_2}{\partial q}=2qP;\; Q=\frac{\partial F_2}{\partial P}=q^2 \qquad \text{(SJ16.03)}$$

74. Let E_S denote the contribution of the surface energy per nucleon in the liquid drop model. The ratio $E_S\left({}^{27}_{13}AI\right) : E_S\left({}^{64}_{30}Zn\right)$ is

a. 2:3 **b.** 4:3

c. 5:3 **d.** 3:2

Ans-b

Let E_S denote the contribution of the surface energy per nucleon in the liquid drop model and is equal to

$$\frac{E_S}{A} \propto 4\pi R^2 \Rightarrow \frac{E_S}{A} = kA^{2/3} \qquad \text{(SJ16.01)}$$

where, we have used,

$$R = R_0 A^{1/3} \qquad \text{(SJ16.02)}$$

The energy of $E_S\left({}^{27}_{13}AI\right) \& E_S\left({}^{64}_{30}Zn\right)$ are

$$\left(\frac{E_S}{A}\right)\left({}^{27}A\ell\right) = k \quad 27^{2/3} \quad ; \quad \left(\frac{E_S}{A}\right)\left({}^{64}Zn\right) = k\ (64)^{2/3} \qquad \text{(SJ16.03a, 03b)}$$

Dividing eq. (SJ16.03a) by eq. (SJ6.03b), we get

$$\frac{E_S\left({}^{27}A_\ell\right)}{E_S\left({}^{64}Z_n\right)} = \frac{A(Zinc)}{A(A\ell)}\left(\frac{27}{64}\right)^{2/3} = \frac{64}{27}\times\frac{9}{16} = \frac{4}{3} \qquad \text{(SJ16.04)}$$

75. In the large hadron collider (LHC), two equal energy proton beams traverse in opposite directions along a circular path of length 27 km. If the total centre of mass energy of a proton-proton pair is 14 TeV, which of the following is the best approximation for the proper time taken by a proton to traverse the entire path?

a. 12 ns **b.** $1.2\mu s$

c. 1.2 ns **d.** $0.12\mu s$

Ans-a

We have,

$$S = 27\,km, E = 27TeV, c = 3\times10^8\,m/s \qquad \text{(SJ16.01)}$$

The time taken to complete one round trip is,

$$T = \frac{S}{c} = \frac{27\times10^3\,m}{3\times10^8\,s} = 90\mu s \qquad \text{(SJ16.02)}$$

The value of relative exponent is given by,

$$p = \gamma m_p c \Rightarrow \gamma = \frac{p}{m_p c} = \frac{cp}{m_p c^2} = \frac{E_p}{m_p c^2} = \frac{7\times10^{12}\,eV}{938MeV} = 7.48\times10^3 \qquad \text{(SJ16.03)}$$

The proper time taken by a proton to traverse the entire path

$$T = \gamma T_0 \Rightarrow T_0 = \frac{T}{\gamma} = \frac{90\,ns}{7.48} = 12ns \qquad \text{(SJ16.04)}$$

H-38

The binding energy B of a nucleus (mass number A and charge Z) is

$$B = a_v A = a_s A^{2/3} - a_{sym}\frac{(A-2Z)^2}{A} - \frac{a_c Z^2}{A^{1/3}} \qquad \text{(SD14.02)}$$

The question has wrongly mentioned $(2Z - A)^2$, instead of $(A-2Z)^2$, as it gives negative answer and from physics point of view a_{sym} term is representing excess of neutron over protons.Thus, we have proceeded with correct energy expression. Differentiating eq. (SD14.02) w.r.t z,and solving for Z_{min}, one obtains

$$\frac{dB}{dz} = 0 \Rightarrow z_{\min} = \frac{A}{2+\left(a_c / 2a_{sym}\right)A^{2/3}} \qquad \text{(SD14.03)}$$

Inserting eq. (SD14.01) in eq. (SD14.03), we obtain the most stable isobar for a nucleus is

$$z_{\min} = \frac{216}{2+(0.75/48)(216)^{2/3}} = 84.3 = 84 \qquad \text{(SD14.04)}$$

June-2014
Part-A

1. **Find the missing letter**

A	**B**	**C**	**D**
F	**I**	**L**	**O**
K	**P**	**U**	**Z**
P	**W**	**D**	**?**

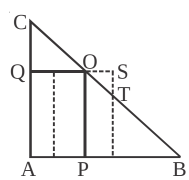
a. P **b.** K

c. J **d.** L

Ans-c
One can easily see that each coulmn contains three constant and a vowel, so in the last coulmn, the last element must be constant as it has already contains vowel 'O'. This constant is J as P, K and L are already prtesent in first three coulmn. The corrected last coulmn is mentioned below.

$$\begin{array}{|c|} D \\ O \\ Z \\ J \end{array}$$

2. **Consider a right-angle tringle ABC where AB = AC = 3. A rectangle APOQ is drawn inside it, as shown such that the height of the rectangle is twice its width. The rectangle is moved horizontlly by a distance 0.2 as shown schematically in the diagram (not to sale).**



What is the ratio $\dfrac{Area \text{ of } \Delta ABC}{Area \text{ of } \Delta OST}$?

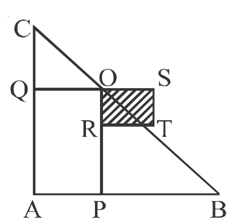
a. 625 **b.** 400

c. 225 **d.** 125

Ans-c
We have,

$$AB = AC = 3, AQ = 2AP, OS = RT = 0.2, OR = ST = 0.2 \qquad \text{(SJ14.01)}$$

The modified diagram after shifting the rectangular is shown below.



One can see that the ΔOST is congruent to ΔORT, and hence these two triangles have the same area. The area of the ΔORT is

$$Area\left(\Delta ORT\right) = \frac{1}{2} \times 0.2 \times 0.2 = 0.02 \qquad \text{(SJ14.02)}$$

On equating the eq. (SJ14.01) with eq. (SJ14.02), we get

$$E_{n_1,n_2} = \frac{\left(n_1^2 + n_2^2\right)\pi^2\hbar^2}{2ma^2} = \frac{5\pi^2\hbar^2}{2ma^2} \Rightarrow n_1^2 + n_2^2 = 5 \qquad \text{(SJ14.03)}$$

and solving for n_1 and n_2, one obtains

$$n_1 = 1, n_2 = 2 \; or \; n_1 = 2, n_2 = 1 \qquad \text{(SJ14.04)}$$

The wave function corresponding to these values, for the fermions, must be antisymmetric i.e.,

$$\psi_{1,2} = \frac{2}{a}\left[\sin\left(\frac{\pi x}{a}\right)\sin\left(\frac{2\pi x}{a}\right) - \sin\left(\frac{2\pi x}{a}\right)\sin\left(\frac{\pi x}{a}\right)\right] \qquad \text{(SJ14.05)}$$

32. A particle of mass m in the potential $V(x, y) = \frac{1}{2}m\omega^2\left(4x^2 + y^2\right)$, is in eigenstate of energy $E = \frac{5}{2}\hbar\omega$. The corresponding un-normalized eigenfunction is

a. $y\exp\left[-\frac{m\omega}{2\hbar}\left(2x^2 + y^2\right)\right]$

b. $x\exp\left[-\frac{m\omega}{2\hbar}\left(2x^2 + y^2\right)\right]$

c. $y\exp\left[-\frac{m\omega}{2\hbar}\left(x^2 + y^2\right)\right]$

d. $xy\exp\left[-\frac{m\omega}{2\hbar}\left(x^2 + y^2\right)\right]$

Ans-a

We have,

$$V(x, y) = \frac{1}{2}m\omega^2\left(4x^2 + y^2\right), \qquad E_{n_x,n_y} = \frac{5}{2}\hbar\omega \qquad \text{(SJ14.01)}$$

The Hamiltonian corresponding to this potential is,

$$\hat{H} = \left(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + \frac{1}{2}m(2\omega)^2 x^2\right) + \left(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial y^2} + \frac{1}{2}m\omega^2 y^2\right) = \hat{H}_{n_x} + \hat{H}_{n_y} \qquad \text{(SJ14.02)}$$

The wave function corresponding to this hamiltonian is

$$\psi(x, y) = \phi_{n_x}(x)\phi_{n_y}(y) \qquad \text{(SJ14.03)}$$

where,

$$\phi_{n_x}(x) = \left(\frac{\gamma}{2^n n!\sqrt{\pi}}\right)^{1/2} H_n\left(\sqrt{\frac{m\omega}{\hbar}}x\right)e^{-m(2\omega)x^2/2\hbar}; \phi_{n_y}(x)$$

$$= \left(\frac{\gamma}{2^n n!\sqrt{\pi}}\right)^{1/2} H_n\left(\sqrt{\frac{m\omega}{\hbar}}y\right)e^{-m\omega y^2/2\hbar} \qquad \text{(SJ14.04)}$$

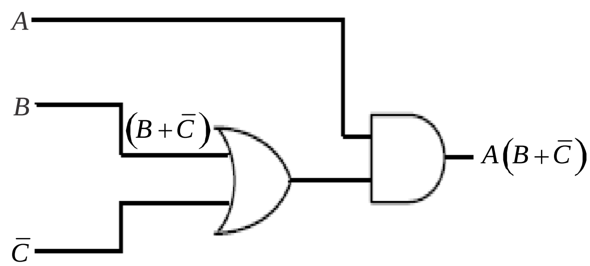
Inserting eq. (SJ14.03) in eq. (SJ14.02), yielding

$$\hat{H}\psi(x, y) = E_{n_x,n_y}\psi(x, y) \Rightarrow E_{n_x,n_y} = \frac{\hat{H}\psi(x, y)}{\psi(x, y)}$$

$$= \frac{\hat{H}_{n_x}\phi(x)}{\phi(x)} + \frac{\hat{H}_{n_y}\phi(y)}{\phi(y)} \Rightarrow E_{n_x,n_y} = E_{n_x} + E_{n_y} \qquad \text{(SJ14.05)}$$

where,

$$E_{n_x} = \left(n_x + \frac{1}{2}\right)(2\hbar\omega), \; E_{n_y} = \left(n_y + \frac{1}{2}\right)\hbar\omega \qquad \text{(SJ14.06)}$$



52. A spectral line due to a transition from an electronic state p to an s state splits into three Zeeman lines in the presence of a strong magnetic filed. At intermediate filed strengths the number of spectral lines is

a. 10 **b.** 3

c. 6 **d.** 9

Ans-d

For p state, the possible values of total angular momentum are

$$J = |\ell - S| \, to \, |\ell + S| = |1-1/2|, |1+1/2| = 1/2, 3/2 \quad \text{(SJ14.01)}$$

Where, we have used

$$\ell = 1, S = 1/2 \quad \text{(SJ14.02)}$$

The number of different energy level corresponding to angular momentum $J = 1/2, 3/2$ are

$$For\ J = 1/2, m_j = -1/2, 1/2\ ;\ For\ J = 3/2, m_j = -3/2, -1/2, 1/2, 3/2 \quad \text{(SJ14.03)}$$

Thus, there are 6 different energy level corresponding to p state. Similarly, for s state, the possible values of total angular momentum are

$$J = |\ell - S| \, to \, |\ell + S| = |0-1/2|, |0+1/2| = 1/2 \quad \text{(SJ14.04)}$$

Where, we have used

$$\ell = 0, S = 1/2 \text{ (SJ14.01)} \quad \text{(SJ14.05)}$$

The number of different energy level corresponding to angular momentum $J = 1/2$ are

$$For\ J = 1/2, m_j = -1/2, 1/2 \quad \text{(SJ14.06)}$$

Thus, there are 2 different energy level corresponding to s state. Thus, the total number of possible transition from P to S state are 12. However, applying the dipole transition relation,

$$\Delta m_j = 0, \pm 1 \quad \text{(SJ14.07)}$$

The transition corresponding to $\Delta m_j = \pm 2$ are not allowed i.e.,

$$(m_j = -3/2) \to (m_j = +1/2), (m_j = 3/2) \to (m_j = -1/2) \quad \text{(SJ14.08)}$$

Hence, the total number of observed spectral line, as shown in the figure, in the presence of strong magnetic field are

$$spectral\ lines = 12 - 2 = 10 \quad \text{(SJ14.09)}$$

53. A particle in the infinite square well

$$\mathbf{V(x)} = \begin{cases} \mathbf{0} & \mathbf{0 < x < a} \\ \infty & \textbf{otherwise} \end{cases}$$

is prepared in a state with the wavefuction

$$\psi(\mathbf{x}) = \begin{cases} \mathbf{A\sin^3\left(\frac{\pi x}{a}\right)} & \mathbf{0 < x < a} \\ \mathbf{0} & \textbf{otherwise} \end{cases}$$

The expectation value of the energy of the particle is

Ans-b

We have,

$$\omega = 10^7 \, rad/s, \sigma = \frac{1}{2\pi} \times 10^6 \, (\Omega m)^{-1}, \frac{I}{I_0} = 1\% \qquad \text{(SJ14.01)}$$

The intensity of an em wave inside the conducting medium is

$$I = I_0 e^{-\alpha x} \qquad \text{(SJ14.02)}$$

Where, α is an attenuation constant and it is defined as

$$\alpha = \frac{2}{skin\ depth} = \frac{2}{\sqrt{2/\mu\omega\sigma}} = \sqrt{2\mu\omega\sigma} \qquad \text{(SJ14.03)}$$

Inserting eq. (SJ14.01) in eq. (SJ14.03), we get

$$\alpha = \sqrt{2\mu\omega\sigma} = \sqrt{2 \times 4\pi \times 10^{-7} \times 10^7 \times \frac{10^6}{2\pi}} \Rightarrow \alpha = 2 \times 10^3 \, m^{-1} \qquad \text{(SJ14.04)}$$

The penetration distance of the em wave inside the conducting medium is

$$I = I_0 e^{-\alpha x} \Rightarrow \ell n\left(\frac{I}{I_0}\right) = -\alpha x \Rightarrow x = -\frac{1}{\alpha} \ell n\left(\frac{I}{I_0}\right) \qquad \text{(SJ14.05)}$$

Inserting eq. (SJ14.04) and eq. (SJ14.01) in eq. (SJ14.05), we obtain the penetration distance as

$$x = -\frac{1}{\alpha} \ell n\left(10^{-2}\right) \Rightarrow x = \frac{2\ell n 10}{\alpha} = \frac{2 \times 2.3 \, m}{2 \times 10^3} = 2.3mm \qquad \text{(SJ14.06)}$$

Note: If attennuatian of only E or B field is asked then

$$\alpha = \frac{1}{skin\, depth}, \qquad \text{(SJ14.07)}$$

and your answer will be 4.6mm but since intensity α square of amplitude of E (or B), one must use the skin depth relation as

$$\alpha = \frac{2}{skin\, depth} \qquad \text{(SJ14.08)}$$

December-2013
Part-A

1. Three fishermen caught fishes and went to sleep. One of them woke up, took away one fish and 1/3rd of the remainder as his share, without other's knowledge. Later, the three of them divided the remained equally. Howmany fishes were caught?

a. 58 **b.** 19

c. 76 **d.** 88

Ans -b

We solve this problem using option:

Let the total fish is 19, the first person takes a fish and 1/3 his share and the number of remaining fishes

$$\text{Re}\,maining\ \ fishes = 19 - 1 - 18/3 = 12 \quad \text{(SD13.01)}$$

These share of each person in the remaining fishes are

$$Share\,of\ \ each\ person\left(\text{Re}\,maining\ \ fish\right) = 12/3 = 4 \quad \text{(SD13.02)}$$

Note: In other option, the remaining fishes are not divisible by 3.

2. What is the arithmetic mean of $\frac{1}{1\times 2}, \frac{1}{2\times 3}, \frac{1}{3\times 4}, \frac{1}{4\times 5}, ..., \frac{1}{100\times 101}$?

a. 0.01 **b.** $\frac{1}{101}$

c. 0.00111 **d.** $\frac{\frac{1}{49\times 50} + \frac{1}{50\times 51}}{2}$

Ans -b

We have

$$\frac{1}{1\times 2}, \frac{1}{2\times 3}, \frac{1}{3\times 4}, \frac{1}{4\times 5}, ..., \frac{1}{100\times 101} \quad \text{(SD13.01)}$$

Rearranging eq. (SJ13.01), we get

$$\left(\frac{1}{1} - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{4}\right) + ... - \frac{1}{101} = 1 - \frac{1}{101} = \frac{100}{101} \quad \text{(SD13.02)}$$

The number of terms in the sequence is 100. So, the arthemetic mean is

$$Arthemetic\ mean = \frac{sum \text{ of all the terms}}{\text{total number of terms}} = \frac{100/101}{100} = \frac{1}{101} \quad \text{(SD13.03)}$$

3. Every time a ball falls to ground, it bounces back to half the height it fell from. A ball is dropped from a height of 1024 cm. The maximum height from the ground to which it can rise after the tenth bounce is

a. 102.4 cm **b.** 1.24 cm

c. 1 cm **d.** 2 cm

Ans - c

Let the initial height be 'x', the height after first drop is

$$h_1 = x/2 \quad \text{(SD13.01)}$$

Similarly, the height after second drop is

$$h_2 = \left(x/2\right)^2 \quad \text{(SD13.02)}$$

Similarly, the height after 10th drop is

$$h_{10} = \left(x/2\right)^{10} = 1024/1024 = 1cm \quad \text{(SD13.03)}$$

a. $T_C > T_B > T_A$ **b.** $T_A > T_C > T_B$

c. $T_B > T_C > T_A$ **d.** $T_B > T_A > T_C$

Ans-b
Let us consider the entropy vs energy curve as shown in the question. Entropy for a system is defined as

$$dS = \frac{dQ}{T} \Rightarrow \frac{1}{T} = \frac{dS}{dQ} = m\left(slope\right) \qquad \text{(SD13.01)}$$

One can see that the slope of phase in the region A is smaller than the slope of phase in region C i.e.,

$$T = \frac{1}{m} \Rightarrow T_A > T_C \; as \; m_1 > m_3 \qquad \text{(SD13.02)}$$

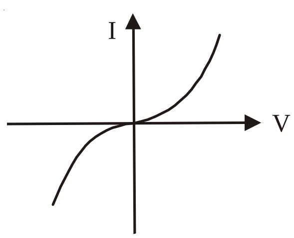
Since the slope in the region B is zero, this does not means that temperature becomes infinity in this region. However, this shows that temperature in this region must be lower than that of in region A and C as entropy remains constant with the increase in energy. Thus the temperature dependence in the three region is given by

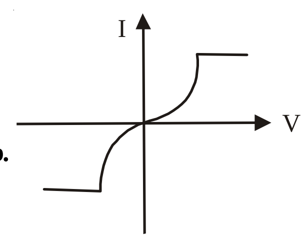
$$T_A > T_C > T_B. \qquad \text{(SD13.03)}$$

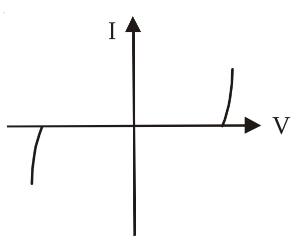
31. The physical phenomenon that cannot be used for memory storage applications is
a. Large variation in magneto resistance as a function of applied magnetic field
b. Variation in magnetization of a ferromagnet as a function of applied magnetic field
c. Variation in polarization of a ferroelectric as a function of applied electric field
d. Variation in resistance of a metal as a function of applied electric field

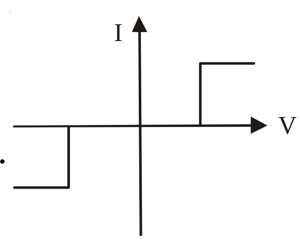
Ans-d
The variation in resistance of a metal as a function of applied electric field can not be used for memory storage function. While in other three cases memory can be stored as they are related to the orientation of spins and change in the resistance of material in the presence of magnetic field.

32. Two identical Zener diodes are placed back to back in series and are connected to a variable DC power supply. The best representation of the I - V characteristics of the circuit is

a.


b.


c.


d.


Ans-c
The V-I characteristics for first diode, the reverse current increases abruptly for the voltage greater than breakdown voltage. The output of the first zener diode is applied as the input to the second zener diode, thus, the second diode reverse the direction of current i.e. the forward current increases abruptly for the voltage greater than the breakdown voltage. These characterstic are correctly shown in the option (c).

60. **The fourier transform of the derivative of the Dirac δ - function, namely $\delta'(x)$, is proportional to**

a. 0 **b.** 1

c. sin k **d.** i k

Ans-d

We have,

$$f(x)=\delta'(x) \quad \text{(SD13.01)}$$

The fourier transform of the derivative of the $\delta-$ function is,

$$F\{\delta'(x)\}=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}e^{ikx}\delta'(x)dx \quad \text{(SD13.02)}$$

Integrating the eq. (SD13.01) using by parts method, we get

$$F\{\delta'(x)\}=\frac{1}{\sqrt{2\pi}}\left[e^{ikx}\left(\int_{-\infty}^{\infty}\delta'(x)dx\right)-ik\int_{-\infty}^{\infty}e^{ikx}\delta(x)dx\right]$$

$$=-\frac{ik}{\sqrt{2\pi}}e^{ik0}\int_{-\infty}^{\infty}\delta(x)dx=-\frac{ik}{\sqrt{2\pi}} \quad \text{(SD13.03)}$$

Where, we have used

$$\int_{-\infty}^{\infty}f(x)\ \delta(x)dx=f(0)\ \text{ and }\int_{-\infty}^{\infty}\delta'(x)dx=0 \quad \text{(SD13.04)}$$

61. **If $A=\hat{i}yz+\hat{j}xz+\hat{k}xy$, then the integral $\oint \vec{A}.\vec{dl}$ (where C is along the perimeter of a rectangular area bounded by $x=0$, $x=a$ and $y=0$, $y=b$) is**

a. $\frac{1}{2}\left(a^3+b^3\right)$ **b.** $\pi\left(ab^2+a^2b\right)$

c. $\pi\left(a^3+b^3\right)$ **d.** 0

Ans-d

We have

$$\vec{A}=\hat{i}\,yz+\hat{j}xz+\hat{k}xy \quad \text{(SD13.01)}$$

Applying the Stoke's theorem, we get

$$\oint \vec{A}\cdot\vec{dl}=\int_s\left(\vec{\nabla}\times\vec{A}\right)\cdot\hat{n}ds \quad \text{(SD13.02)}$$

The curl of eq. (SD13.01) is given

$$\vec{\nabla}\times\vec{A}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial y}\\ yz & xz & xy\end{vmatrix}=\hat{i}\left(\frac{\partial(xy)}{\partial y}-\frac{\partial(xz)}{\partial z}\right)-\hat{j}\left(\frac{\partial(xy)}{\partial x}-\frac{\partial(yz)}{\partial z}\right)+\hat{k}\left(\frac{\partial(xz)}{\partial x}-\frac{\partial(yz)}{\partial y}\right)=0 \quad \text{(SD13.03)}$$

Inserting eq. (SD13.03) in eq. (SD13.02), we obtain

$$\oint_C \vec{A}\cdot\vec{dl}=\int_s(0)\cdot\hat{n}ds=0 \quad \text{(SD13.04)}$$

62. **Consider an $n\times n(n>1)$ matrix A, in which A_{ij} is the product of the indicies i and j (namely A_{ij} = ij). The matrix A**

a. has one degenerate eigenvalue with degeneracy $(n-1)$
b. has two degenerate eigenvalues with degeneracies 2 and $(n-2)$
c. has one degenerate eigenvalue with degeneracy n
d. Does not have any degeenrate eigenvalue

where N is the number of particle, q is the position, p is the momentum and E is the energy of the system. The energy of the particle is

$$E = \frac{p^2}{2m} + \frac{1}{2}kr^2 \quad \text{(SD13.03)}$$

Inserting eq. (SD13.03) in eq. (SD13.02), we get

$$Z_1 = \frac{1}{h^3}\int e^{-\beta\frac{p^2}{2m}} d^3p \int e^{-\beta\frac{kr^2}{2}} d^3r$$

$$= \frac{(4\pi)^2}{h^3}\int_0^\infty p^2 e^{-\frac{\beta P^2}{2m}} dp \int_0^\infty r^2 e^{-\frac{\beta kr^2}{2}} dr = \frac{\pi}{2}\frac{(4\pi)^2}{h^3\beta^3}\left(\frac{m}{k}\right)^{3/2} \quad \text{(SD13.04)}$$

The integral in eq. (SD13.) are solved in following manner: Let us consider the first integral

$$I = \int_0^\infty p^2 e^{-\frac{\beta p^2}{2m}} dp = \frac{1}{2}\left(\frac{2m}{\beta}\right)^{3/2} \int_0^\infty t^{3/2-1} e^{-t} dt$$

$$= \frac{1}{2}\left(\frac{2m}{\beta}\right)^{3/2} \Gamma(3/2) = \frac{\sqrt{\pi}}{4}\left(\frac{2m}{\beta}\right)^{3/2} \quad \text{(SD13.05)}$$

where, we have used,

$$\frac{\beta p^2}{2m} = t \Rightarrow p = \sqrt{\frac{2m}{\beta}}\, t^{1/2} \Rightarrow dp$$

$$= \frac{1}{2}\sqrt{\frac{2m}{\beta}}\, t^{-1/2} dt \Rightarrow p^2 dp = \frac{1}{2}\left(\frac{2m}{\beta}\right)^{3/2} t^{1/2} dt \quad \text{(SD13.06)}$$

Similarly, the second integral is evaluated as

$$I_2 = \int_0^\infty r^2 e^{-\frac{\beta kr^2}{2}} dr = \frac{1}{2}\left(\frac{2}{\beta k}\right)^{3/2} \int_0^\infty t^{3/2-1} e^{-t} dt$$

$$= \frac{1}{2}\left(\frac{2}{\beta k}\right)^{3/2} \Gamma(3/2) = \frac{\sqrt{\pi}}{4}\left(\frac{2}{\beta k}\right)^{3/2} \quad \text{(SD13.07)}$$

where, we have used,

$$\frac{\beta kr^2}{2} = t \Rightarrow r = \sqrt{\frac{2}{\beta k}}\, t^{1/2} \Rightarrow dr$$

$$= \frac{1}{2}\sqrt{\frac{2}{\beta k}}\, t^{-1/2} dt \Rightarrow r^2 dr = \frac{1}{2}\left(\frac{2}{\beta k}\right)^{3/2} t^{1/2} dt \quad \text{(SD13.08)}$$

The internal energy of the system is defined as

$$U = -\frac{\partial}{\partial\beta}\ln Z_N = -\frac{\partial}{\partial\beta}\ln(Z_1)^N = -N\frac{\partial}{\partial\beta}(\ln A - 3\ln\beta) = \frac{3N}{\beta} = 3NKT \quad \text{(SD13.09)}$$

Where, we have used

$$Z_1 = \frac{\pi}{2}\frac{(4\pi)^2}{h^3}\left(\frac{m}{k}\right)^{3/2}\beta^{-3} \Rightarrow \ln Z_1 = \ln A - 3\ln\beta;\; A = \frac{\pi}{2}\frac{(4\pi)^2}{h^3}\left(\frac{m}{k}\right)^{3/2} \quad \text{(SD13.10)}$$

75. A child makes a random walk on a square lattice of lattice constant a taking a step in the north, east, south, or west directions with probabilities 0.255, 0.255, 0.245 and 0.245, respectively. After a large number of steps, N, the expected position of the child with respect to the starting point is at a distance

a. $\sqrt{2}\times10^{-2}\,Na$ in the north - east direction

b. $\sqrt{2N}\times10^{-2}$ a in the north - east direction

c. $2\sqrt{2}\times10^{-2}\,Na$ in the south - west direction

d. 0

Ans-a

June 2013
Part-A

1. **There is an equilateral triangle in the XY-plane with its centre at the origin. The distance of its sides from the origin is 3.5 cm. The area of its circumcircle in cm^2 is**

a. 38.5 **b.** 49

c. 63.65 **d.** 154

Ans-d

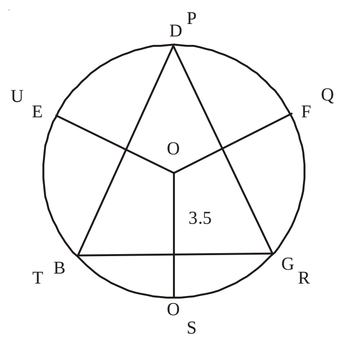
The radius of the circle is

$$OS = r = 2\times 3.5 = 7\,cm \qquad \text{(SJ13.01)}$$

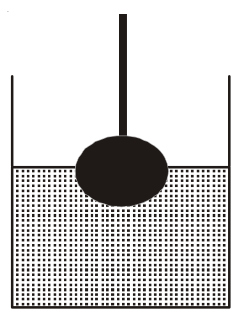
The area of the circle is

$$A = \pi r^2 = \pi \times 7^2 = \frac{22}{7}\times 7\times 7 = 154\ cm^2 \qquad \text{(SJ13.02)}$$

The figure shows the triangle inside the triangle is given below.



2. **A sphere of iron of radius R/2 fixed to one end of a string was lowered into water in a cylindrical container of base radius R to keep exactly half the sphere dipped. The rise in the level of water in the container will be**



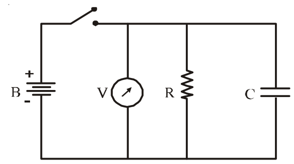
a. R/3 **b.** 49

c. 63.65 **d.** 154

Ans-d

When a sphere of iron is dipped inside the cylindrical container such that it is exactly half dipped in the water, the water in the container will rise equal to the half volume displaced by the sphere i.e.,

$$V_{Cylind} = (1/2)V_{sph} \Rightarrow \pi R^2 h = \frac{1}{2}\left(\frac{4}{3}\pi \times \left(\frac{R}{2}\right)^3\right) \Rightarrow \pi R^2 h = \frac{\pi R^3}{12} \Rightarrow h = \frac{R}{12} \qquad \text{(SJ13.01)}$$



a. $10^9\,\Omega$

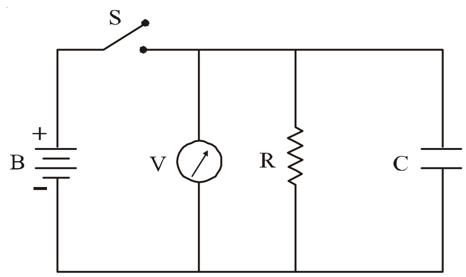
b. $10^8\,\Omega$

c. $10^7\,\Omega$

d. $10^6\,\Omega$

Ans-b

We have,



Let i is the total current supplied by the battery, now applying the KVL in the resistance and capacitor circuit i.e.,

$$\left(C\frac{dV}{dt}\right)X_c+\left(C\frac{dV}{dt}X_C-i\right)R=0\Rightarrow C\frac{dV}{dt}\left(X_C+R\right)$$

$$=iR=V\Rightarrow\frac{dV}{V}=\frac{dt}{R+X_C} \qquad \text{(SJ13.01)}$$

On integrating the eq. (SJ13.01), we get

$$\ell nV\Big|_{150}^{15}=\frac{1}{C\left(R+X_C\right)}t\Rightarrow\left|\ln\frac{15}{150}\right|=\left|-1\right|=\left|\frac{t}{C\left(R+X_C\right)}\right|$$

$$=\left|\frac{j\omega t}{\left(1+j\omega RC\right)}\right|=\frac{1000\omega}{\sqrt{1+\omega^2R^2C^2}}=\frac{1000}{RC\sqrt{2}} \qquad \text{(SJ13.02)}$$

The value of the resistance is

$$R=\frac{1000}{C\sqrt{2}}=\frac{1000}{5\times10^{-6}\times\sqrt{2}}=1.414\times10^8=10^8\,\Omega \qquad \text{(SJ13.03)}$$

30. The approximation $\cos\theta\approx1$ is valid up to 3 decimal placed as long as $|\theta|$ is less than: (take $180°/\pi \approx 57.29°$)

a. 1.28°

b. 1.81°

c. 3.28°

d. 4.01°

Ans-c

We have,

$$\cos\theta=1 \qquad \text{(SJ13.01)}$$

Applying the taylor's expansion of $\cos\theta$ and considering the first 3 terms only in eq.(SJ13.01), we obtain

$$1-\frac{\theta^2}{2!}+\frac{\theta^2}{4!}...=1\Rightarrow\frac{\theta^4}{4!}-\frac{\theta^2}{2!}=0\Rightarrow\theta^2\left[\theta^2-12\right]$$

$$=0\Rightarrow\theta=0,\ \theta=\sqrt{12}=3.46° \qquad \text{(SJ13.02)}$$

Let us evaluate the value of each of these inner product separately. So the value of first term of inner product is

$$\left\langle \psi \,|\, \frac{d^2\psi}{dx^2} \right\rangle = \left\langle \psi \left| -2b + 4b^2x^2 \right| \psi(x) \right\rangle = -2b\langle \psi \,|\, \psi \rangle + 4b^2 \left\langle \psi \left| x^2 \right| \psi \right\rangle \qquad \text{(SJ13.06)}$$

or

$$\left\langle \psi \,|\, \frac{d^2\psi}{dx^2} \right\rangle = -2b + 4b^2 \left(\frac{2b}{\pi}\right)^{1/2} \int_{-\infty}^{\infty} x^2 e^{-2b^2x^2}\, dx$$

$$= -2b + 4b^2 \left(\frac{2b}{\pi}\right)^{1/2} 2\frac{\sqrt{\pi}}{4}\frac{1}{(2b)^{3/2}} = -2b + 4b^2\frac{1}{4b} = -b \qquad \text{(SJ13.07)}$$

Similarly, the value second term of inner product is

$$\left\langle \psi(x) \left| \delta(x) \right| \psi(x) \right\rangle = \left(\frac{2b}{\pi}\right)^{1/2} \int_{-\infty}^{\infty} \delta(x) e^{-2bx^2}\, dx$$

$$= \left(\frac{2b}{\pi}\right)^{1/2} e^{-2bx^2}\Big|_{x=0} = \left(\frac{2b}{\pi}\right)^{1/2} \qquad \text{(SJ13.08)}$$

substituting eq. (SJ13.07) and eq. (SJ13.08) in eq. (SJ13.05), we get

$$E(b) = \frac{\hbar^2 b}{2m} - \alpha\left(\frac{2b}{\pi}\right)^{1/2} \qquad \text{(SJ13.09)}$$

The value of variation parameter b is obtained by differentiating $E(b)$ given in eq. (SJ13.09) w.r.t b i.e

$$\frac{dE(b)}{db} = \frac{\hbar^2}{2m} - \alpha\left(\frac{2}{\pi}\right)^{1/2}\frac{1}{2}b^{-1/2} = 0 \Rightarrow b = \frac{2m^2\alpha^4}{\pi\hbar^4} \qquad \text{(SJ13.10)}$$

Substituting eq. (SJ13.10) in eq. (SJ13.09), we have

$$E_{\min} = -\frac{m\alpha^2}{\pi\hbar^2} \qquad \text{(SJ13.11)}$$

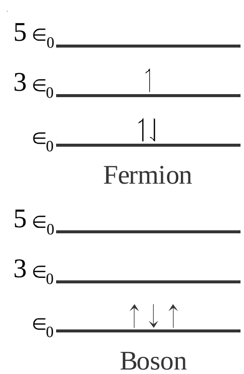
54. Consider two diferent systems each with three identical non - interacting particles. Both have single particle states with energies $\varepsilon_0, 3\varepsilon_0$ and $5\varepsilon_0, (\varepsilon_0 > 0)$. One system is populated by spin 1/2 fermions and the other by bosons. What is the value of $E_F - E_\beta$, where E_F and E_B are the ground state energies of the fermionic and bosonic systems respectively?

a. $6\varepsilon_0$ **b.** $2\varepsilon_0$

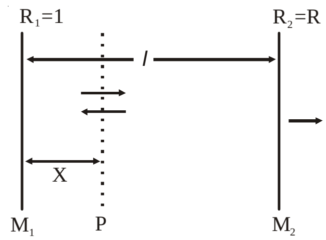
c. $4\varepsilon_0$ **d.** ε_0

Ans-b

The ground state of the fermion and boson particle are shown in the figure given below:



74. Consider the laser resonator cavity shown in the figure. If I_1 is the intensity of the radiation at mirror M_1 and α is the gain coefficient of the medium between the mirrors, then the energy densityof photons in the plane P at a distance x from M_1 is



a. $(I_1/c)e^{i\alpha x}$

b. $(I_1/c)e^{\alpha x}$

c. $(I_1/c)(e^{\alpha x}+e^{-\alpha x})$

d. $(I_1/c)e^{\alpha x}$

Ans-c

75. A spin 1/2 particle A undergoes the decay, $A \rightarrow B + C + D$; where it is known that B and C are also spin 1/2 particles. The complete set of allowed values of the spin of the particle D is

a. $\frac{1}{2},1,\frac{3}{2},2,\frac{5}{2},3,....$

b. 0, 1

c. 1/2 only

d. $\frac{1}{2},\frac{3}{2},\frac{5}{2},\frac{7}{2},.....$

Ans-d

We have,

$$A \rightarrow B + C + D \quad \text{(SJ13.01)}$$

also the spins of the particles A, B and C is 1/2 respectively. The possible values of D is given by

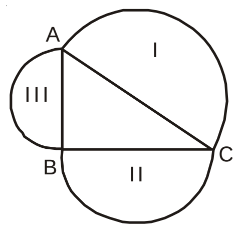
$$A \rightarrow B + C + D \Rightarrow \frac{1}{2} \rightarrow \frac{1}{2}+\frac{1}{2}+D \Rightarrow 1+D \rightarrow \frac{1}{2} \quad \text{(SJ13.02)}$$

In order to satisfy the eq. (SJ13.02), the particle D can not have any integer values, so the possible values of D are

$$D = \frac{1}{2},\frac{3}{2},\frac{5}{2},\frac{7}{2},..... \quad \text{(SJ13.03)}$$

December - 2012
Part-A

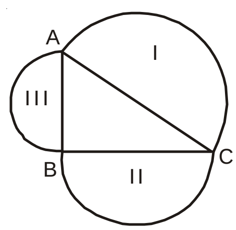
1. In the figure below, angle $ABC = \pi/2$. I, II, III are the areas of semicircles on the sides opposite angles B, A and C, respectively. Which of the following always true?



a. $II^2 + III^2 = I^2$ **b.** $II + III = I$

c. $II^2 + III^2 > I^2$ **d.** $II + III < I$

Ans-b
We have,



Applying Pythagoras theoram in ΔABC, we obtain

$$AC^2 = AB^2 + BC^2 \quad \text{(SD12.01)}$$

The area of the semicircle for the region I is

$$I = \frac{\pi(AC/2)^2}{2} = \frac{\pi AC^2}{8} \quad \text{(SD12.02)}$$

The area of the semicircle for the region II is

$$II = \frac{\pi(BC/2)^2}{2} = \frac{\pi BC^2}{8} \quad \text{(SD12.03)}$$

The area of the semicircle for the region III is

$$III = \frac{\pi(AB/2)^2}{2} = \frac{\pi AB^2}{8} \quad \text{(SD12.04)}$$

Now adding eq. (SD12.03) and eq. (SD12.04), we get

$$II + III = \frac{\pi BC^2}{8} + \frac{\pi AB^2}{8} = \frac{\pi}{8}\left(AB^2 + BC^2\right) = \frac{\pi}{8}AC^2 = I \Rightarrow II + III = I \quad \text{(SD12.05)}$$

2. A peacock perched on the top of a12 m high tree spots a snake moving towards its hole at the base of the tree from a distance equal to thrice the height of the tree. The peacock files towards the snake in a straight line and they both move the same speed. At what distance the base of the tree will the peacock catch the snake?

a. 16m **b.** 18 m

c. 14 m **d.** 12 m

28. **A binary star system consists of two stars S_1 and S_2, with masses m and 2m respectively separated by a distance r. If both S_1 and S_2 individually follow circular orbits around the centre of mass with instantaneous speeds v_1 and v_2 respectively, the speeds ratio v_1/v_2 is**

a. $\sqrt{2}$ **b.** 1

c. 1/2 **d.** 2

Ans-d

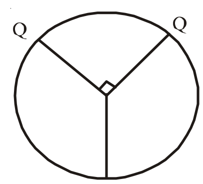
Let the m and 2m are the masses of the two stars moving with velocities v_1 & v_2 respectively. The angular momentum for the stars, moving in the circular orbit, is always conserved i.e.,

$$L_1 = L_2 \Rightarrow mv_1 r = 2\,mv_2 r \qquad \text{(SD12.01)}$$

From the eq. (SD12.01), one, therefore, obtains the ratio of velocities for the two stars as

$$v_1 : v_2 = 2:1 \qquad \text{(SD12.02)}$$

29. **Three charges are located on the circumference of radius R as shown in the figure below. The two charges Q subtend an angle 90° at the centre of the circle. The charge q is symmetrically placed with respect to the charges Q. If the electric field at the centre of the circle is zero, what is the magnitude of Q?**

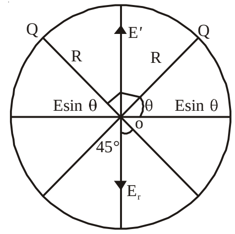


a. $\frac{q}{\sqrt{2}}$ **b.** $\sqrt{2}q$

c. 2q **d.** 4q

Ans-a

The three charges are placed at the circumference of the radius R. The two charges Q are placed such that they subtends an angle 90° at the centre and the third charge q is placed symmetrically w.r.t Q as shown in figure given below.



The electric field produced by the two charges at centre 'O' is evaluated as follows:The electric field produced by the charge Q in component form is written as: the horizontal component of electric field at point 'O' is

$$E_H = \frac{1}{4\pi \in_0} \frac{Q}{R^2} \cos 45^\circ = \frac{1}{4\sqrt{2}\pi \in_0} \frac{Q}{R^2} \qquad \text{(SD12.01)}$$

Similarly, the perpendicular component of electric field at point 'O' is

$$E_\perp = \frac{1}{4\pi \in_0} \frac{Q}{R^2} \sin 45^\circ = \frac{1}{4\sqrt{2}\pi \in_0} \frac{Q}{R^2} \qquad \text{(SD12.02)}$$

The perpendicular components of two charges are equal and opposite, and hence the cancel out. The resultant electric field due to charges 'Q' at point 'O' is equal to the sum of the horizontal component of electric field i.e.,

$$E_r = 2E_H = \frac{2}{4\sqrt{2}\pi \in_0} \frac{Q}{R^2} \qquad \text{(SD12.03)}$$

a. $\frac{\mu_0 Kt}{2}\hat{j}$

b. $-\frac{\mu_0 Kz}{2c}\hat{j}$

c. $-\frac{\mu_0 K}{2c}(ct-z)\hat{i}$

d. $-\frac{\mu_0 K}{2c}(ct-z)\hat{j}$

Ans-d

We have,

$$\vec{A}=\frac{\mu_0 k}{4c}(ct-z)^2\,\hat{i} \qquad \text{(SD12.01)}$$

The magnetic field in terms of vector potential is expressed as

$$\vec{B}=\vec{\nabla}\times\vec{A} \qquad \text{(SD12.02)}$$

Inserting eq. (SD12.02) in eq. (SD12.01), the magnetic field is

$$\vec{B}=\vec{\nabla}\times\vec{A}=\begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}=\begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{\mu_0 k}{4c}(ct-z)^2 & 0 & 0 \end{vmatrix}=-\frac{\mu_0 k}{2c}(ct-z)\,\hat{j} \qquad \text{(SD12.03)}$$

56. When a charged particle emits electromagnetic radiation, the electricfield $\vec{E}$ and the Poynting vector $\vec{S}=\frac{1}{\mu_0}\vec{E}\times\vec{B}$ at a large distance r from the emitter vary as $1/r^n$ and $1/r^m$ respectively. Which of the following choices for n and m are correct?

a. $n=1$ and $m=1$

b. $n=2$ and $m=2$

c. $n=1$ and $m=2$

d. $n=2$ and $m=4$

Ans-c

The variation of the electric field and poynting vector with distance ‘r’ is

$$E\propto\frac{1}{r^n}\,;\; S\propto\frac{1}{r^m} \qquad \text{(SD12.01)}$$

In radiation zone, the electric field, magnetic field and poynting vector are given by

$$E\propto\frac{1}{r}\,;\; B\propto\frac{1}{r}\,;\; S=\frac{1}{\mu_0}\vec{E}\times\vec{B}\propto\frac{1}{r^2} \qquad \text{(SD12.02)}$$

On equating eq. (SD12.01) with eq. (SD12.02), we obtain the value of constant n and m as

$$n=1, m=2 \qquad \text{(SD12.03)}$$

57. The energies in the ground states and first excited state of a particle of mass $m=1/2$ in a potential $V(x)$ are -4 and -1, respectively, (in units in which $\hbar=1$). If the correspondig wavefunctions are related by $\psi_1(x)=\psi_0(x)\,sin\,hx$, then the ground state eigenfunction is

a. $\psi_0(x)=\sqrt{\text{sech x}}$

b. $\psi_0(x)=\sec hx$

c. $\psi_0(x)=\sec h^2 x$

d. $\psi_0(x)=\sec h^3 x$

Ans-c

The ground state wavefunction and its corresponding energy, in the unit $(\hbar=1)$, in the potential V(x) for a particle of mass (m = 1/2)is

$$\psi_0=\psi_0(x),\; E_0=-4, \qquad \text{(SD12.01)}$$

where, the concentration of free eletrons in fcc structure is

$$n = \frac{no.of\ atoms \times no.\ free\, e^-}{\text{volume of unit cell}} = \frac{4\times 1}{a^3} = \frac{4}{a^3} \qquad \text{(SD12.04)}$$

73. The muon has mass 105 MeV/c² and mean lifetime 2.2μs in its rest frame. The mean distance traversed by a muon of energy 315 *MeV/c²* before decaying is approximately

a. 3×10^5 km | **b.** 2.2 cm

c. $6.6\mu m$ | **d.** 1.98 km

Ans-d

We have,

$$m_0 = 105\,MeV/c^2, \Delta\tau = 2.2\,\mu s, m = 315\,MeV/c^2 \qquad \text{(SD12.01)}$$

The lorentz factor is obtained from the expression of the relativistic mass of the particle i.e.,

$$m = \frac{m_0}{\sqrt{1-v^2/c^2}} = \gamma m_0, \Rightarrow 315\,MeV = 105\,MeV \Rightarrow \gamma = 3 \qquad \text{(SD12.02)}$$

Now, the dilated time Δt taken by the particle before decaying is

$$\Delta t = \gamma\Delta\tau = 3\times 2.2 = 6.6\,\mu s \qquad \text{(SD12.03)}$$

The distance traversed by the muon before decaying is,

$$d = c\times\Delta t = 3\times 10^5 \times 6.6\times 10^{-6}\ km = 1.98\ km \qquad \text{(SD12.04)}$$

74. Consider the following particles; the proton p, the neutron n, the neutral pion π^0 and the delta resonance Δ^+. When ordered in terms of decreasing lifetime, the correct arrangement is as follows:

a. π^0, n, p, Δ^+ | **b.** p, n, Δ^+, π^0

c. p, n, π^0, Δ^+ | **d.** Δ^+, n, π^0, p

Ans-c

The correct specification of the particles are given below.

Particles	Rest Mass (MeV/c^2)	Life time	
P	93.83	stable	
	n	939.3	925
	π°	135.0	8×10^{-17}
	Δ^+	-	-

So, the correct arrangement in decreasing order of lifetime is, $p, n, \pi^\circ, \Delta^+$.

75. The single particle energy difference between the p-orbitals (i.e, $p_{3/2}$ and $p_{1/2}$) of the nucleus $^{114}_{50}$Sn is 3*MeV*. The energy difference between the states in its 1 f orbital is

a. – 7MeV | **b.** 7 MeV

c. 5MeV | **d.** – 5MeV

Ans-b

June - 2012
Part-A

1. **In still air, fragrance of a burning incense stick will be smelt by an observer quickest when the experiment is carried out at**

a. Low altitude and high air temperature

b. high altitude and low air temperature

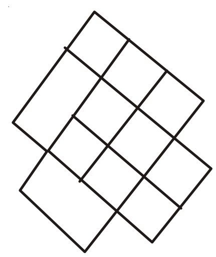
c. low altitude and low air temperature

d. hig altitude and high air temperature

Ans-d

Diffusion of air accelerates at high altitude and high temperature of air.

2. **How many squares are there in this figure?**



a. 9

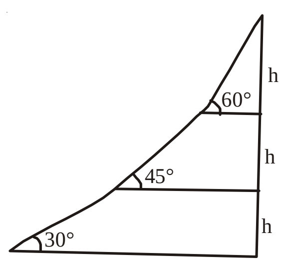
b. 14

c. 15

d. 17

Ans-c

3. **A mountain road has 3 sections of different slopes as shown. What is the average slope m of the entire climb?**



a. **1**

b. $(1/3) < m < (1/2)$

c. $1 < m < \sqrt{3}$

d. $(1/\sqrt{3}) < m < 1$

Ans-d

The average slpoes of the entire climb, as shown in the figure, is

$$\text{Average slope } (m) = \frac{m_1 + m_2 + m_3}{3} \qquad \text{(SJ12.01)}$$

One can divide the climb into 3 triangles and determine the slopes of each triangle seprately. The slope of the ΔABC is

$$m_1 = \tan 30° = \frac{1}{\sqrt{3}} \qquad \text{(SJ12.02)}$$

36. A particle of mass m is in a cubic box of size a. The potential inside the box $(0 \le x < a, 0 \le y < a, 0 \le z < a)$ is zero and infinite outside. If the particle is in an eigenstate of energy $E = \frac{14\pi^2\hbar^2}{2ma^2}$, it wavefunction is

a. $\psi = \left(\frac{2}{a}\right)^{3/2} \sin\frac{3\pi x}{a} \sin\frac{5\pi y}{a} \sin\frac{6\pi z}{a}$

b. $\psi = \left(\frac{2}{a}\right)^{3/2} \sin\frac{7\pi}{x} \sin\frac{4\pi y}{a} \sin\frac{3\pi z}{a}$

c. $\psi = \left(\frac{2}{a}\right)^{3/2} \sin\frac{4\pi x}{a} \sin\frac{8\pi y}{a} \sin\frac{2\pi z}{a}$

d. $\psi = \left(\frac{2}{a}\right)^{3/2} \sin\frac{\pi x}{a} \sin\frac{2\pi y}{a} \sin\frac{3\pi z}{a}$

Ans-d

A energy of the particle of mass m confined in a cubical box of size is

$$E = \frac{\hbar^2\pi^2}{2ma^2}\left(n_x^2 + n_y^2 + n_z^2\right) \quad \text{(SJ12.01)}$$

and its corresponding wave function are

$$\psi = \left(\frac{2}{a}\right)^{3/2} \sin\left(\frac{n_x \pi x}{a}\right) \sin\left(\frac{n_y \pi y}{a}\right) \cdot \sin\left(\frac{n_z \pi y}{a}\right) \quad \text{(SJ12.02)}$$

The particle of an eigenstate of energy is given by,

$$E = \frac{14\pi^2\hbar^2}{2ma^2}; \quad \text{(SJ12.03)}$$

where a is the size of cubic box. On comapring eq. (SJ12.01) with eq. (SJ12.02), we get

$$(1,2,3),(1,3,2),(3,1,2),(2,1,3),(3,2,1),(2,3,1) \quad \text{(SJ12.04)}$$

The wavefunction corresponding to the given energy eigenvalue can take anyone of the degenrate state i.e

$$\psi = \left(\frac{2}{a}\right)^{3/2} \sin\left(\frac{\pi x}{a}\right) \cdot \sin\left(\frac{2\pi y}{a}\right) \cdot \sin\left(\frac{3\pi z}{a}\right) \quad \text{(SJ12.05)}$$

37. Let ψ_{nlm} denote the eigenfunction of Hamiltonian for a spherically symmetric potential $V(r)$. The wavefunction $\psi = \frac{1}{4}\left[\psi_{210} + \sqrt{5}\psi_{21\text{-}1} + \sqrt{10}\psi_{213}\right]$ is an eigenfunction only of

a. H, L^2 and L_z

b. H and L_z

c. H and L^2

d. L^2 and L_z

Ans-c

We have

$$\psi = \frac{1}{4}\left[\psi_{210} + \sqrt{5}\psi_{21-1} + \sqrt{10}\psi_{211}\right] \quad \text{(SJ12.01)}$$

Any function is said to be an eigenfunction of an operator A if it satisfies the condition

$$A\psi = a\psi \quad \text{(SJ12.02)}$$

Now applying the Hamiltonian operator $\hat{H}$ on the wave function ψ is

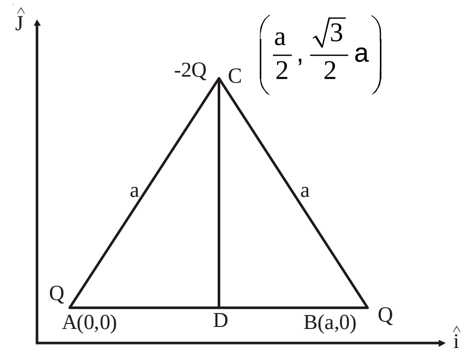
$$H\psi = \frac{1}{4}\left[H\psi_{210} + \sqrt{5}H\psi_{21-1} + \sqrt{10}H\psi_{211}\right]$$

$$= \frac{1}{4}\left[E_2\psi_{210} + \sqrt{5}E_2\psi_{21-1} + \sqrt{10}E_2\psi_{211}\right]$$

$$\hat{H}\psi = \frac{E_2}{4}\left[\psi_{210} + \sqrt{5}\psi_{21-1} + \sqrt{10}\psi_{211}\right] = E_2\psi \quad \text{(SJ12.03)}$$

Ans-c

The Charges Q, Q and -2Q are placed on the vertices of an equilateral triangle ABC of sides of length as shown in the given figure. The co-ordinates of these charges are shown in the figure.



The dipole moment for a system of discrete charges are obtained by the relation

$$\hat{p}(dipole\,moment) = \sum_i q_i\,(with\,sign\,) \times \sum \vec{r}\,(position\,of\,\,charge\,at\,that\,\,point) \qquad \text{(SJ12.01)}$$

Since the total charge for this system of charges is zero, the dipole moment for this system is independent of the co-ordinate system or the origin chosen at any charge. Let us now consider the origin at co-ordinate A, the dipole moment of this system is

$$\hat{p} = q_a r_a + q_b r_b + q_c r_c = (+Q)(0) + (+Q)(a\hat{i}) + (-2Q)\left(a/2\hat{i} + \sqrt{3}/2\hat{j}\right)$$

$$= -\sqrt{3}aQ\,\hat{j} \qquad \text{(SJ12.02)}$$

59. **The vector potential $\vec{\mathbf{A}}$ due to a magnetic moment $\vec{m}$ at a point $\vec{r}$ is given by $\mathbf{A} = \frac{\mathbf{\vec{m}} \times \mathbf{\vec{r}}}{\mathbf{r}^3}$. If $\mathbf{\vec{m}}$ is directed along the positive z-axis, the x-component of the magnetic field, at the point $\mathbf{\vec{r}}$, is**

a. $\frac{3myz}{r^5}$

b. $-\frac{3mxy}{r^5}$

c. $-\frac{3mxz}{r^5}$

d. $\frac{3m\left(z^2 - xy\right)}{r^5}$

Ans-c

The vector potential $\vec{A}$ due to a magnetic moment $\vec{m}$ at a point $\vec{r}$ is given by

$$\vec{A} = \frac{\vec{m} \times \vec{r}}{r^3} \qquad \text{(SJ12.01)}$$

If $\vec{m}$ is directed along the positive z-axis, the magnetic field, at the point $\vec{r}$, is

$$\vec{B} = \vec{\nabla} \times \vec{A} = \vec{\nabla} \times \frac{(\vec{m} \times \vec{r})}{r^3} = r^{-3}\left[\vec{\nabla}.(\vec{m} \times \vec{r})\right] + \nabla r^{-3} \times (\vec{m} \times \vec{r}) \qquad \text{(SJ12.02)}$$

Or

$$\vec{B} = r^{-3}(2\vec{m}) - \frac{3}{r^5}\left[(\vec{r} \cdot \vec{r})\vec{m} - (\vec{m} \cdot \vec{r})\vec{r}\right]$$

$$= r^{-3}(2\vec{m}) - r^{-3}(3\vec{m}) + \frac{3}{r^5}\left[(\vec{m} \cdot \vec{r}).\vec{r}\right] \qquad \text{(SJ12.03)}$$

Or

$$\vec{B} = -\frac{\vec{m}}{r^3} + \frac{3}{r^5}\left[(\vec{m} \cdot \vec{r}).\vec{r}\right] \qquad \text{(SJ12.04)}$$

$$k = \frac{2n\pi}{L}; n = 1, 2, 3, ... \& \ g = \frac{2\pi m}{L}, n = 1, 2, 3, ...\qquad \text{(SJ12.06)}$$

73. The ground state of $^{207}_{82}$Pb nucleus has spin parity $J^P = \frac{1}{2}$, while the first excited state has $J^P = \frac{5^-}{2}$. The electromagnetic radiation emitted when the nucleus makes a transition from the first excited state to the ground state are

a. E_2 and E_3 **b.** M_2 and E_3

c. E_2 and M_3 **d.** M_2 and M_3

Ans-c

The ground state of $^{207}_{82}Pb$ nucleus has spin parity $J^p = 1/2$, while the first excited state has $J^p = (5/2)^-$. The condition for E type transistion, there is no change in parity i.e parity before and after transition remains same. Similarly, for M type transition, there occurs change in parity i.e., even parity changes to odd parity and vice versa. The change in angular momentum is

$$\Delta J = |J_1 + J_2|, |J_1 - J_2| = 3, 2 \qquad \text{(SJ12.01)}$$

The condition of parity is given by

$$P = (-1)^{\Delta j} \qquad \text{(SJ12.02)}$$

One can see that for $\Delta J = 2$, there is no change in parity, thus it leads to E_2. For $\Delta J = 3$, there is change in parity M_ℓ type, thus it leads to M_3.

74. The dominant interactions underlying the following process

P. $K^- + p \to \Sigma^- + \pi^+$

Q. $\mu\mu^- + \mu^+ \to K^- + K^+$

R. $\Sigma^+ \to p + \pi^0$ are

a. P: strong, Q : electromagnetic and R : weak
b. P : strong, Q : weak and R : weak
c. P: weak, Q : electromagnetic and R : strong
d. P: weak, Q: electromagnetic and R : weak

Ans-c

75. If a Higgs boson of mass m_H with a speed $\beta = v/c$ decays into a pair photons, then the invariant mass of the photon pair is
[Note: The invariant mass of a system of two particles, with four - momenta p_1 and p_2 is $(p_1 + p_2)^2$]

a. βm_H **b.** m_H

c. $m_H\sqrt{1-\beta^2}$ **d.** $\beta m_H / \sqrt{1-\beta^2}$

Ans-b
The mass of Higgs boson is equal to invariant mass of the photon is i.e. m_H.

December - 2011
Part-A

1. **The most abudnant element by mass in the human body is**

a. carbon **b.** hydrogen

c. calcium **d.** oxygen

Ans-d

Oxygen is the most abundant element by mass in the human body.

2. **A number system consists of digits 0, 1, 2, 3, 4 and 5. What is the decimal equivalent of 15 in this number system?**

a. 15 **b.** 13

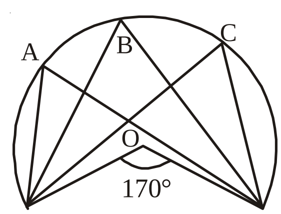
c. 11 **d.** 12

Ans-c

The decimal equivalent of 15 in the given number system is $(15)_6 = 1\times 6^1 + 5\times 6^0 = 11$.

3. **A segment of a circle (slightly greater than a semicircle, whose centre is O) is given below. Identify the correct statement regarding the three angles A, B and C.**

a. A is equal to B but not equal to C
b. A, B and C are equal and have a value of 85°
c. A, B and C are unequal
d. A, B and C are each equal to 95°



Ans-b

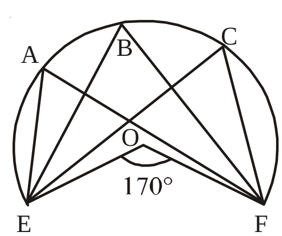
The angle subtended by an arc at the circumference is half of the angle subtended by an arc at the centre.

$$\angle POQ = 2\angle PAQ = 2\angle PBQ = 2\angle PCQ \quad \text{(SD11.01)}$$

So, the angle at the point A, B and C are

$$\angle PAQ = \frac{1}{2}\angle POQ = \frac{1}{2}\times 170° = 85° \quad \text{(SD11.02)}$$

Thus, A, B, O and C are equal and have a value at 85°.



4. **After bubbling air through pure water ($pH = 7.0$), its pH decreased. Which of the following is responsible for the pH change?**

a. Nitrogen **b.** Carbon dioxide

c. Oxygen **d.** Helium

The second integral in eq. (SD11.04) is evaluated as follows:

$$I_2 = \int_0^R \left(1-e^{-r/R}\right)\vec{\nabla}^2\left(\frac{1}{r}\right)dV = -4\pi\int_0^r \left(1-e^{-r/R}\right)\delta(r)\,dV = -4\pi\left(1-e^0\right) = 0 \qquad \text{(SD11.05)}$$

where, we have used

$$\vec{\nabla}^2\left(\frac{1}{r}\right) = -4\pi\delta(r) \qquad \text{(SD11.06)}$$

The first integral in eq. (SD11.04) is evaluated as follows:

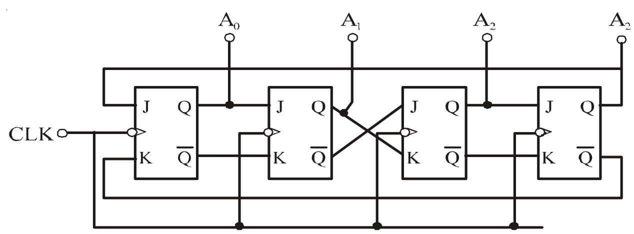
$$I_1 = \int_0^R \left(\vec{\nabla}\left(1-e^{-r/R}\right)\right)\frac{\vec{r}}{r^3}dV = \frac{1}{R}\int_0^R e^{-r/R}\vec{\nabla}(r)\frac{\vec{r}}{r^3}dV \qquad \text{(SD11.07)}$$

$$I = \frac{1}{R}\int_0^R e^{-r/R}\frac{\vec{r}}{r^3}.\frac{\vec{r}}{r^3}4\pi r^2 dr = \frac{4\pi}{R}\int_0^R e^{-r/R}dr = 4\pi\left(1-e^{-1}\right) \qquad \text{(SD11.08)}$$

Substituting eq. (SD11.05) and eq. (SD11.08) in eq. (SD11.04), we obtain

$$Q = \epsilon_0 \int_0^R \vec{\nabla}.\vec{E}\,dV = \epsilon_0\,\alpha\left[4\pi\left(1-e^{-1}\right)+0\right] = 4\pi\,\epsilon_0\,\alpha\left(1-e^{-1}\right) \qquad \text{(SD11.09)}$$

29. A counter consist of four flip - flops connected as shown in the figure.



If the counter is initialized as $A_0A_1A_2A_3$ = 0110, the state after the next clock pulse is

a. 1000 **b.** 0001

c. 0011 **d.** 1100

Ans-b

Since the counter, given in question, is synchronous, all the flip flop are clocked simultaneously. The state of counter is stablized at the state mentioned below

$$\begin{matrix} A_0 & A_1 & A_2 & A_3 \\ 0 & 1 & 1 & 0 \\ Q_0 & Q_1 & Q_2 & Q_3 \\ 0 & 1 & 1 & 0 \end{matrix} \qquad \text{(SD11.01)}$$

The input applied at each J-K flip flop, see the truth table, is

$$J_0 = Q_3\ J_1 = Q_0\ J_2 = \dot{Q}_1\ J_3 = Q_2$$

$$K_0 = \dot{Q}_3\ K_1 = \dot{Q}_0\ K_2 = Q_1\ K_3 = \dot{Q}_2 \qquad \text{(SD11.02)}$$

The truth table of J-K flip flop is

J	K	$Output\ (Q_n)$
0	0	$No\ chnage\ (Q_n)$
0	1	0
1	0	1
1	1	$\dot{Q}_n$

Thus, the output corresponding to each flip flops are

$$J_0 = 0\ J_1 = 0\ J_2 = 0\ J_3 = 1$$

$$K_0 = 1\ K_1 = 1\ K_2 = 1\ K_3 = 0 \qquad \text{(SD11.03)}$$

c. $\left[\left(x-\frac{\pi}{4}\right)-\frac{1}{3!}\left(x-\frac{\pi}{4}\right)^3 .....\right]$

d. $\frac{1}{\sqrt{2}}\left[1-x+\frac{x^2}{2!}-\frac{x^3}{31}.....\right]$

Ans-b

We have

$$f(x)=\sin x \qquad \text{(SD11.01)}$$

The taylor series expansion of the function f(x) at $x=a$ is

$$f(x)=f(a)+\frac{(x-a)}{1!}f'(a)+\frac{(x-a)^2}{2!}f''(a)+.... \qquad \text{(SD11.02)}$$

Inserting eq. (SD11.01) in eq. (SD11.02) and taking $a=\pi/4$, we get

$$f(x)=\left[\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}\left(x-\frac{\pi}{4}\right)-\frac{1}{2!\sqrt{2}}\left(x-\frac{\pi}{4}\right)^2 ....\right]$$

$$=\frac{1}{\sqrt{2}}\left[1+\left(x-\frac{\pi}{4}\right)-\frac{1}{2!}\left(x-\frac{\pi}{4}\right)^2 ....\right] \qquad \text{(SD11.03)}$$

64. A one dimensional chain consists of a set of N each of length a. When stretched by a load, each rod can align either parallel or perpendicular to the length of the chain. The energy of a rod is $-\varepsilon$ when aligend parallel to the length of the chain and is $+\varepsilon$ when perpendicular to it. When the chain is in thermal equilibrium at temeprature T, its average length is:

a. Na/2

b. Na

c. $Na/\left(1+e^{-2\varepsilon/k_BT}\right)$

d. $Na\left(1+e^{-2\varepsilon/k_BT}\right)$

Ans-c

A one dimensional chain consists of a set of N each of length a. When stretched by a load, each rod has energy $-\varepsilon$ when align parallel to the length of the chain and has $+\varepsilon$ when align perpendicular to the length of the chain. The partition function for such a system is

$$z=\sum_r e^{-\beta E_r}=e^{-\beta\varepsilon}+e^{\beta\varepsilon} \qquad \text{(SD11.01)}$$

The system will be in equilibrium whenever its energy becomes minimum, which happens only when the rod are aligned parallel to the chain of the length. The probabilty of finding the chain in that states is

$$P=\frac{e^{-\beta\varepsilon}}{z}=\frac{e^{-\beta\varepsilon}}{e^{-\beta\varepsilon}+e^{\beta\varepsilon}} \qquad \text{(SD11.02)}$$

The mean length of the segment is,

$$\langle \ell \rangle = \ell P=\frac{Nae^{\beta\varepsilon}}{e^{-\beta\varepsilon}+e^{\beta\varepsilon}}=\frac{Na}{1+e^{2\varepsilon/k_BT}};\ \ell = Na \qquad \text{(SD11.03)}$$

65. If the hyperfine interaction in an atom is given by $H = a\vec{S}_e \times \vec{S}_p$, where $\vec{S}_e$ and $\vec{S}_p$ denote the electron and proton spins, respectively, the splitting between the $^3S_1$ and $^1S_0$ state is

a. $a\hbar^2/\sqrt{2}$

b. $a\hbar^2$

c. $a\hbar^2/2$

d. $2a\hbar^2$

Ans-b

The hyperfine interaction in an atom is

$$H=a\vec{S}_e\cdot\vec{S}_p \qquad \text{(SD11.01)}$$

The electric field at a distance $r < ct$ from the infinite straight long wire is

$$\vec{E} = -\frac{\partial \vec{A}}{\partial t} = -\frac{\partial}{\partial}\frac{\mu_0 I}{2\pi}\ln\left[\frac{1}{r}\left(ct + \sqrt{c^2t^2 - r^2}\right)\right]\hat{k}$$

$$= -\frac{\mu_0 Ic}{2\pi\sqrt{c^2t^2 - r^2}}\hat{k}. \qquad \text{(SD11.02)}$$

75. Monochromatic light of wavelength 660 nm and intensity $100\,\text{mW / cm}^2\text{mW / cm}^2$ falls on a solar cell of area 30 3cm^2. The conversion efficiency of the solar cell is 10%. If each converted photon results in an electron hole pair, what is themaximum circuit current supplied by the solar cell? (Take $h = 6.6\times10^{-34}$ Js, $c = 3\times10^8$ m / s and $e = 1.6\times10^{-19}$ C).

a. 160 mA
b. 320 mA
c. 1600 mA
d. 3200 mA

Ans-a

We have

$$Q.E = 10\%,\ A = 30\%,\ I = 100\,mW / cm^2, \lambda = 660\,nm. \qquad \text{(SD11.01)}$$

The quantum efficiency of the solar cell is defined as

$$Q.E = \frac{1240}{\lambda(nm)}\times R = \frac{1240}{\lambda(nm)}\times\frac{I_{op}}{P_{in}(W)} \qquad \text{(SD11.02)}$$

Where responsivity is defined as

$$R = \frac{I_{op}}{P_{in}(W)}; \qquad \text{(SD11.03)}$$

Inserting the eq. (SD11.01) in eq. (SD11.02), the maximum current supplied by the solar cell is

$$\frac{1240}{660}\times\frac{I_{op}}{3} = 0.1 \Rightarrow I_{op} = \frac{0.1\times660\times3}{1240} = 160\,mA \qquad \text{(SD11.04)}$$

June - 2011
Part-A

1. **A physiological disorder X always leads to the disorder Y. However, disorder Y may occur by itself. A population shows 4% incidence of disorder Y. Which of the following inferences is valid?**
a. 4% of the population suffers from both X and Y
b. Less than 4% of the population suffers from X
c. At least 4% of the population suffers from X.
d. There is no incidence of X in the given population

Ans-b
It is given that disorder Y occurs either through disorder X or by itself. Since, incidence of the disorder Y in the population is 4%, one can inferred that there are less than 4% people of population who are suffering from disorder X.

2. **Exposing an organism to a certain chemical can change nucleotide bases in a gene, causing mutation. In one such mutated organism if a protein had only 70% of the primary amino acid sequence, which of the following is likely?**
a. Mutation broke the protein.
b. The organism could not make amino acids
c. Mutation created a terminator codon
d. The gene was not transcribed.

Ans-b
Terminator codon is produced through mutation.

3. **The speed of a car increaes every minute as shown in the following table:**

Time (Minutes)	1	2	3	–	–	24	25
Speed (Meter/Second)	1.5	3.0	4.5	–	–	36.0	37.5

The speed at the end of the 19th minute would be
a. 26.5
b. 28.0
c. 27.0
d. 28.5

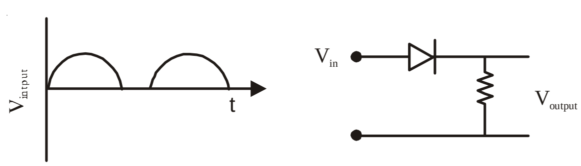
Ans-d
The velocity of the car is in the arithmetic progression with respect to time. In the AP series, the n-th term is given by

$$V_{nth} = a + (n-1)d, \qquad \text{(SJ11.01)}$$

Where d represents the common difference between two term and a is the first term of the A.P series. According to the question, a = 1.5 m/s, d = 3.0 - 1.5 = 1.5 and n = 19. Substituting these value in above equation, the velocity of the car at 19 min is given by

$$V_{19} = 1.5 + (19-1)1.5 = 28.5\, m/s \qquad \text{(SJ11.02)}$$

4. **If V_{input} is applied to the circuit shown, the output would be**



The expectation value of eq. (SJ11.01) is

$$\langle u(x)\rangle = u_0 \langle x\rangle \qquad \text{(SJ11.02)}$$

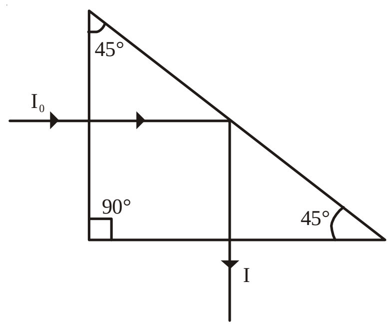
According to equi partition theorem, we have

$$\langle V\rangle = k_B T \qquad \text{(SJ11.03)}$$

On equating eq. (SJ11.02) with eq. (SJ11.03), we get the value of $\langle x\rangle$ as

$$\langle u(x)\rangle = \langle V\rangle \Rightarrow \langle x\rangle = \frac{k_B T}{u_0} \qquad \text{(SJ11.04)}$$

31. Circularly polarized light with intensity I_0 is incident normally on a glass prism as shown in the figure. The index of refraction of glass is 1.5. The intensity I of light emerging from the prism is



a. I_0

b. $0.96\ I_0$

c. $0.92 I_0$

d. $0.881 I_0$

Ans-b

We have

$$n_1 = 1.5, n_2 = 1.0 \qquad \text{(SJ11.01)}$$

The transmission coefficient is defined as

$$T = \frac{4n_1 n_2}{(n_1 + n_2)^2} \qquad \text{(SJ11.02)}$$

where n_1 & n_2 are the refractive indices in two medium. Inserting eq. (SJ11.01) in eq. (SJ11.02), we obtain

$$T = \frac{4\times 1.5}{(1+1.5)^2} = \frac{6}{(2.5)^2} = \frac{6.00}{6.25} = 0.96. \qquad \text{(SJ11.03)}$$

So, the intensity of light emerging from the prism is

$$I = TI_0 = 0.96 I_0 \qquad \text{(SJ11.04)}$$

32. The acceleration due to gravity (g) on the surface of Earth is approximately 2.6 times that on the surface of Mars. Given that the radius of Mars is about one half the radius of Earth, the ratio of the escape velocity on Earth to that on Mars is approximately

a. 1.1

b. 1.3

c. 2.3

d. 5.2

Ans-c

The escape velocity is defined as

$$v_e = \sqrt{\frac{2GM}{r}} = \sqrt{2gr} \qquad \text{(SJ11.01)}$$

a. 750 μW **b.** 675 μW

c. 250 μW **d.** 225 μW

Q. The generated photo - current for a quantum efficiency of unity will be

a. 360 μA **b.** 400 μA

c. 133 μA **d.** 120 μA

P. Ans-a

We have

$$P_{inc} = 1\,mW = 1000\,\mu W \;;\; P_{ref} = 1\%\,P_{inc}$$

$$= 10\,\mu W; t = (\ln 4)\,\mu m; P_0 = 990\,\mu W \qquad \text{(SJ11.01)}$$

The power transmitted in the semiconductor is

$$P_{trans} = P_0\, e^{-\alpha t} \qquad \text{(SJ11.02)}$$

Where, P_0 is the power transmission by the semiconductor in unexcited state, t is the thickness and α is the absorption co-efficient. Substituting eq. (SJ11.01) in eq. (SJ11.02), we obtain

$$P_t = P_0 e^{-\alpha t} = 990 e^{-10^4 \times \ln 4 \times 10^{-4}} = \frac{990}{4} = 247.5\,\mu m \qquad \text{(SJ11.03)}$$

So, the power absorbed by the semiconductor is

$$P_i = P_r + P_t + P_a \Rightarrow P_a = P_i - P_t - P_r = 1000 - 247.5 - 10 \cong 750\,mW. \qquad \text{(SJ11.04)}$$

Q. Ans-b

We have

$$\lambda = 660\,nm;\; Q.E = 1, P_{trans} = 750\,mW \qquad \text{(SJ11.01)}$$

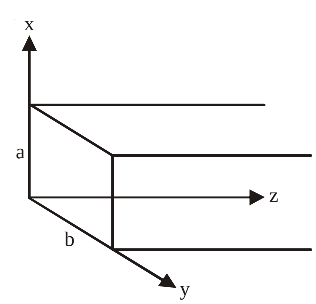
The quantum efficiency of the semiconductor is defined as

$$Q.E = \frac{R \times 1240}{\lambda(nm)} = \frac{I_{OP}(A) \times 1240}{P_{abs}(W) \times \lambda(nm)};\; R = \frac{I_{OP}(A)}{P_{in}(W)} \qquad \text{(SJ11.02)}$$

Substituting eq. (SJ11.01) in eq. (SJ11.02), we obtain

$$I_{OP} = \frac{750 \times 660}{1240} \cong 400\,\mu A \qquad \text{(SJ11.03)}$$

54. **The magnetic field of the TE_{11} mode of a rectangular waveguide of dimensions $a \times b$ as shown in the figure is given by $H_z = H_0 \cos(0.3\pi x)\cos(0.4\pi y)$, where x and y are in cm.**



P. The dimensions of the waveguide are

a. $a = 3.33\,cm, b = 2.50\,cm$ **b.** $a = 0.40\,cm, b = 0.30\,cm$

c. $a = 0.80\,cm, b = 0.60\,cm$ **d.** $\boldsymbol{a = 1.66\,cm, b = 1.25\,cm}$

The number of particles N presents in all excited state in BE condensation is

$$N = \int_0^\infty D(\in) F_{B.E}(\in) d\in = \frac{4\pi V}{h^3}\frac{1}{sa^{3/s}}\int_0^\infty \frac{\varepsilon^{\frac{3-s}{s}} d\in}{e^{\beta\in}-1} \qquad \text{(SJ11.04)}$$

One can see that from eq. (SJ11.04) that $(s \to 0) \Rightarrow (N \to \infty)$. Further, the number of particle becomes zero when $\in \to 0$, for $s = 1 \& s = 3$. Therefore, the range of s for which the system undergoes a BE condensation at non zero temperature is given by $1 < s < 3$.

65. Two gravitating bodies A and B with masses m_A and m_B, respectively, are moving in circular orbit. Assume that $m_B >> m_A$ and let the radius of the orbit of body A be R_A. If the body A is losing mass adibatically. Its orbital radius R_A is proportional to

a. $1/m_A$

b. $1/m_A^2$

c. m_A

d. m_A^2

Ans-b

Since the body is loosing mass through adiabatic process, the angular momentum of the body remains conserved i.e.

$$L_A = L_B = constant \Rightarrow m_A V_A R_A = m_B V_B R_B = k \qquad \text{(SJ11.01)}$$

Where L_A & L_B are the angular momentum of the bodies A and B. The velocity of the body A, from eq. (SJ11.01) is

$$V_A = k / m_A R_A; \qquad \text{(SJ11.02)}$$

where, k is constant. The gravitational force acting between two bodies A and B moving in a circular orbit is

$$F = \frac{Gm_B m_A}{R_A^2}, \qquad \text{(SJ11.03)}$$

The centripital force required for the circular motion of the body is

$$F_{cp} = \frac{m_A v_A^2}{R_A} \qquad \text{(SJ11.04)}$$

On equating the eq. (SJ11.03) with eq. (SJ11.04), we obtain the variation of the orbital radius R_A as

$$\frac{m_A v_A^2}{R_A} = \frac{Gm_B m_A}{R_A^2} \Rightarrow \left(\frac{k}{m_A R_A}\right)^2 = \frac{Gm_B}{R_A} \Rightarrow R_A \propto \frac{1}{m_A^2} \qquad \text{(SJ11.05)}$$